The nucleon-nucleon system in chiral effective theory

Daniel Phillips
Ohio University

Research supported by the US Department of Energy

Plan

- χET for nuclear forces: the proposal
- Leading order for S waves
- Higher orders and higher partial waves
- Elastic electron-deuteron scattering

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  Weinberg (1990), apply χPT to V:

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

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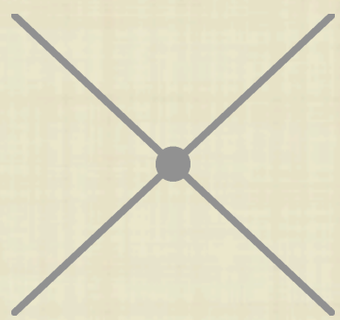
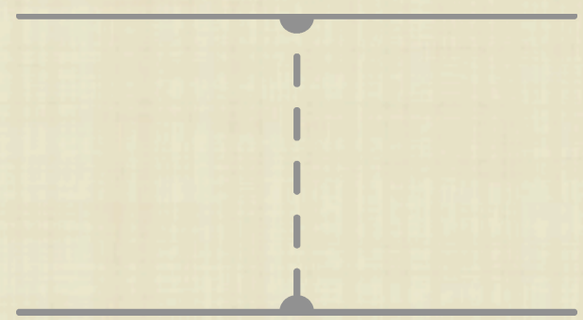
$V^{(0)} =$  $+$  ;

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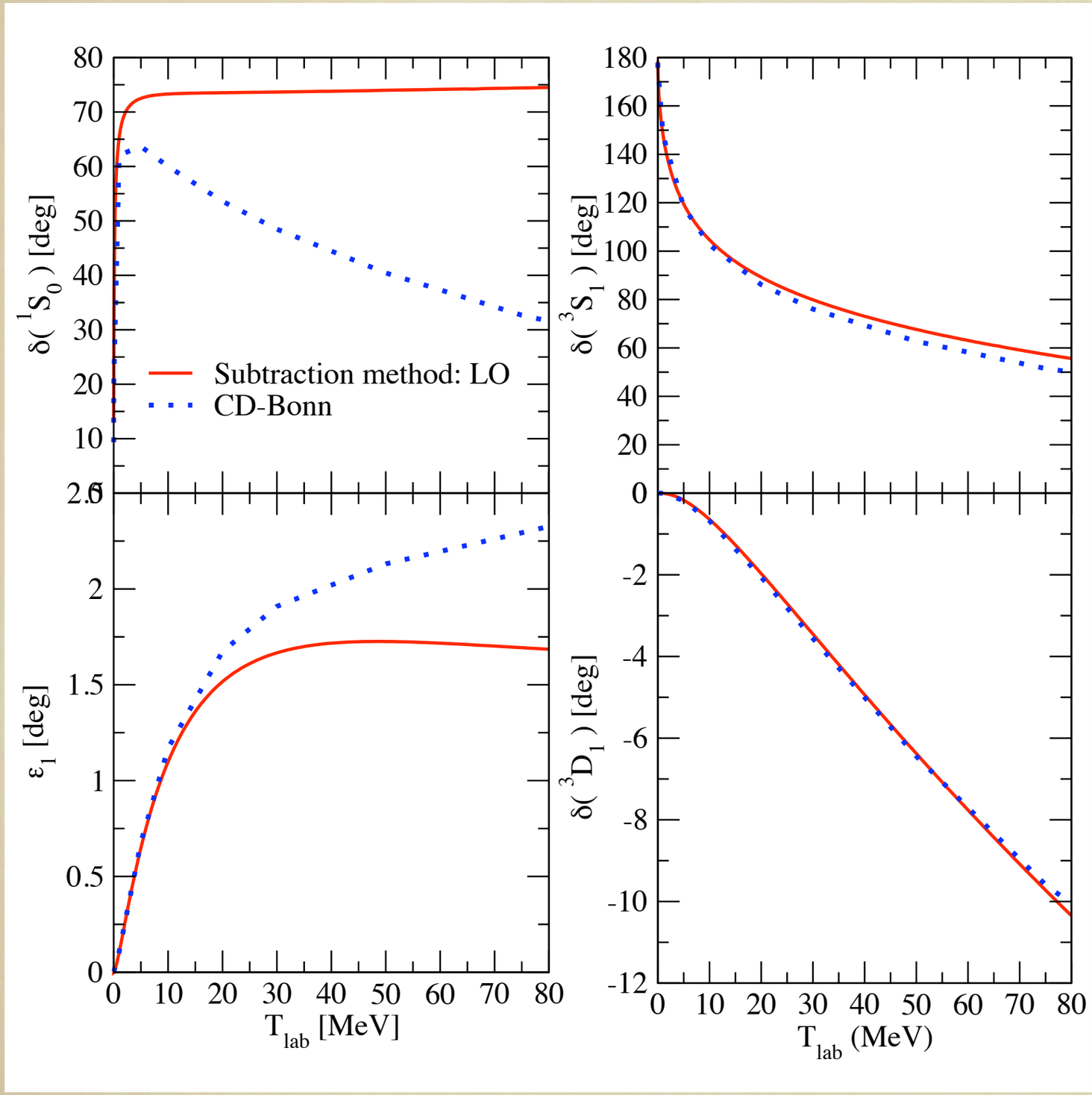
- Different Λ, different short-distance physics. Different predictions?

Results: S waves at leading order

Yang, Elster, Phillips, PRC 77, 014002 (2007)



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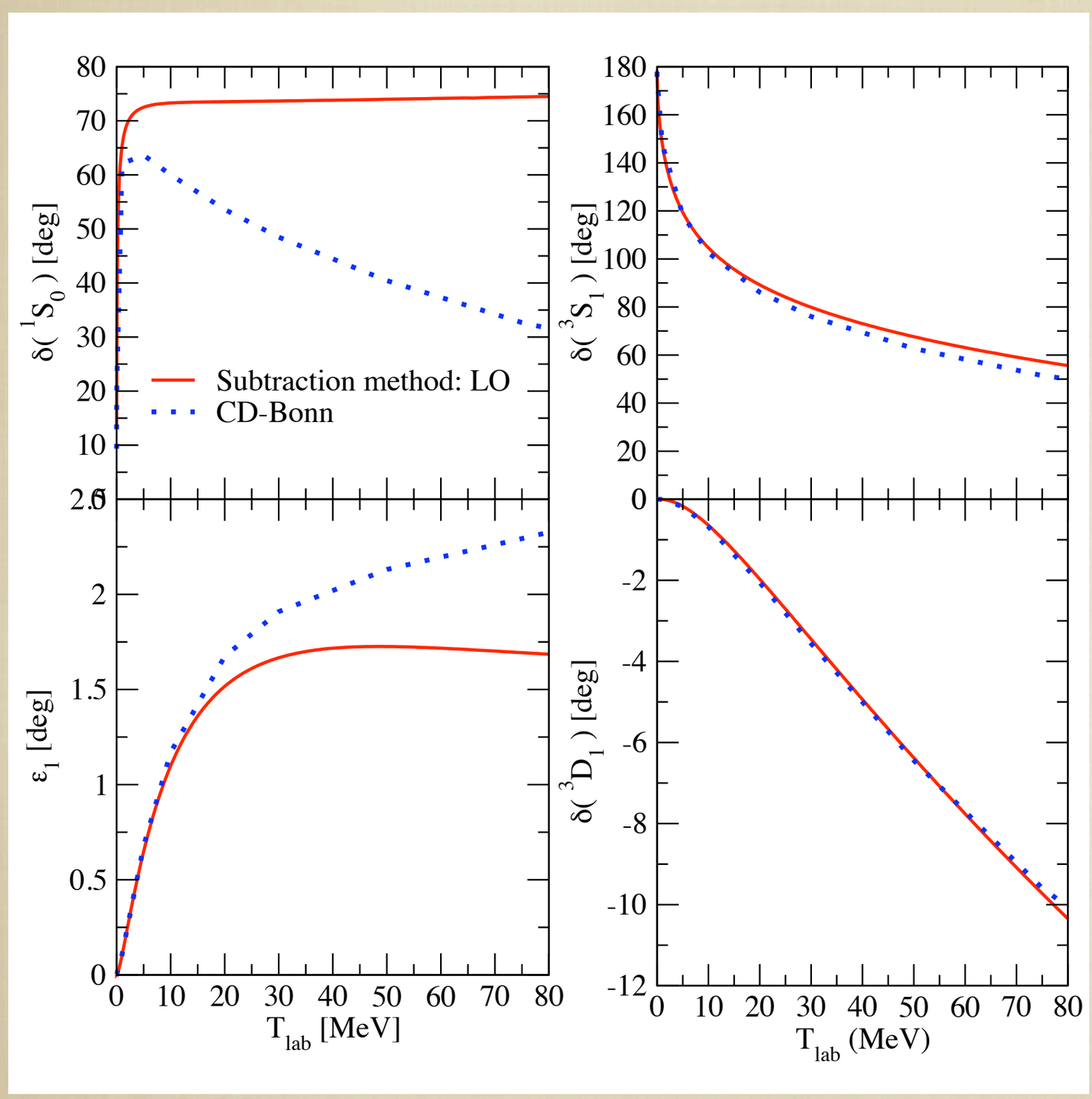

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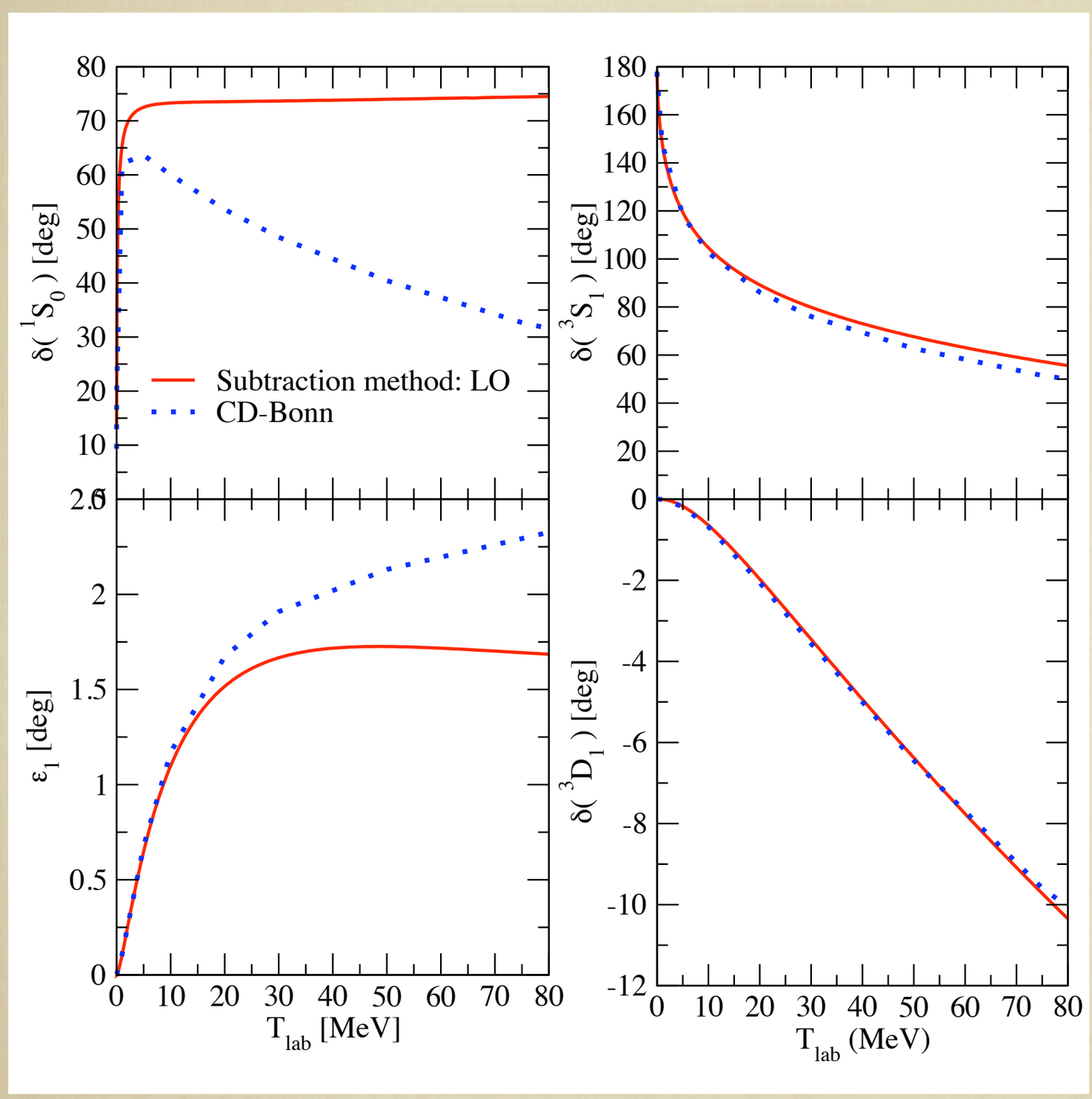

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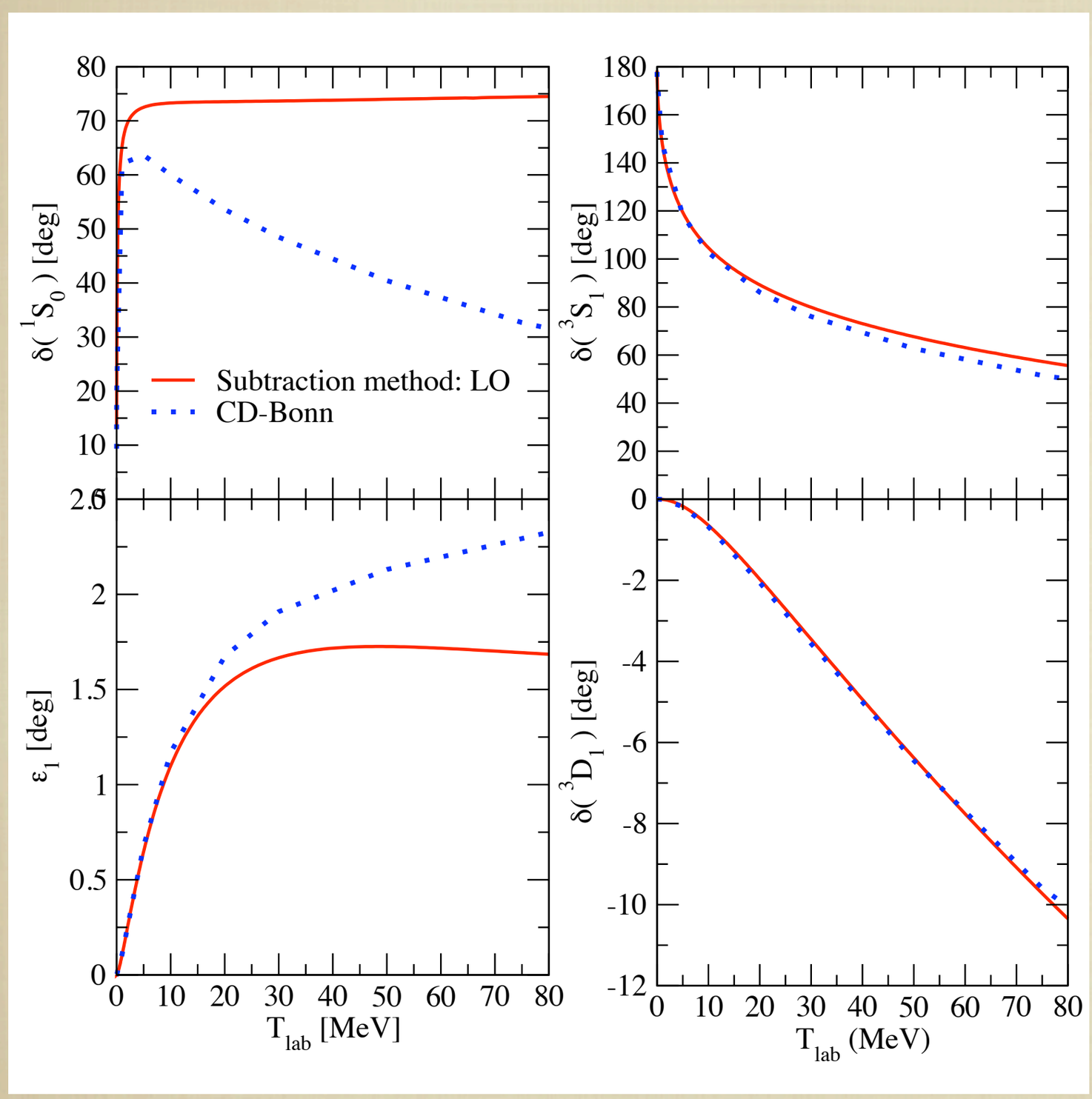

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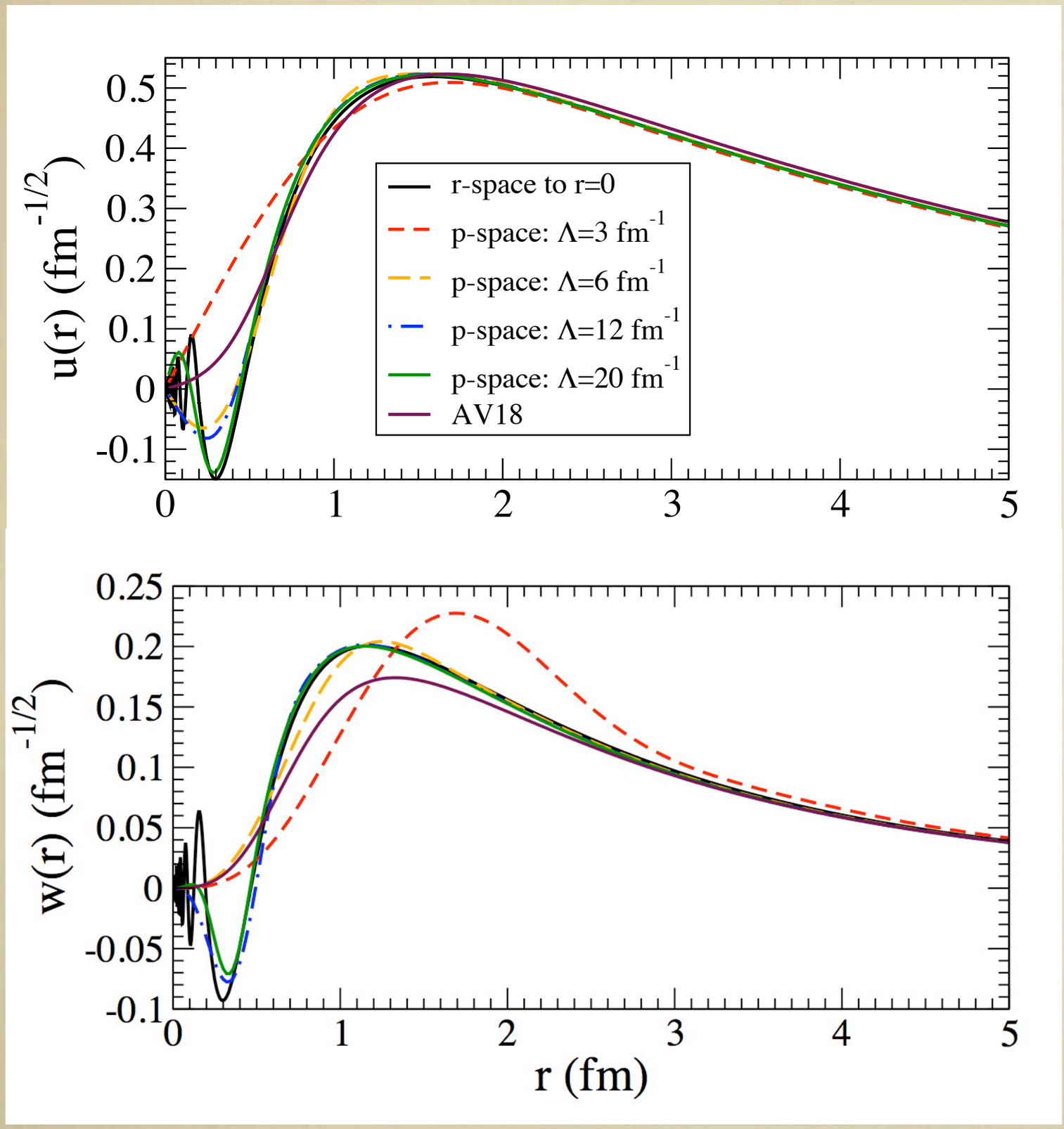
- 2π exchange needed, especially in $^1S_0$

χET deuteron wave functions at leading order

Pavon Valderrama, Nogga, Ruiz Arriola,DP, EPJA 36, 315 (2008)



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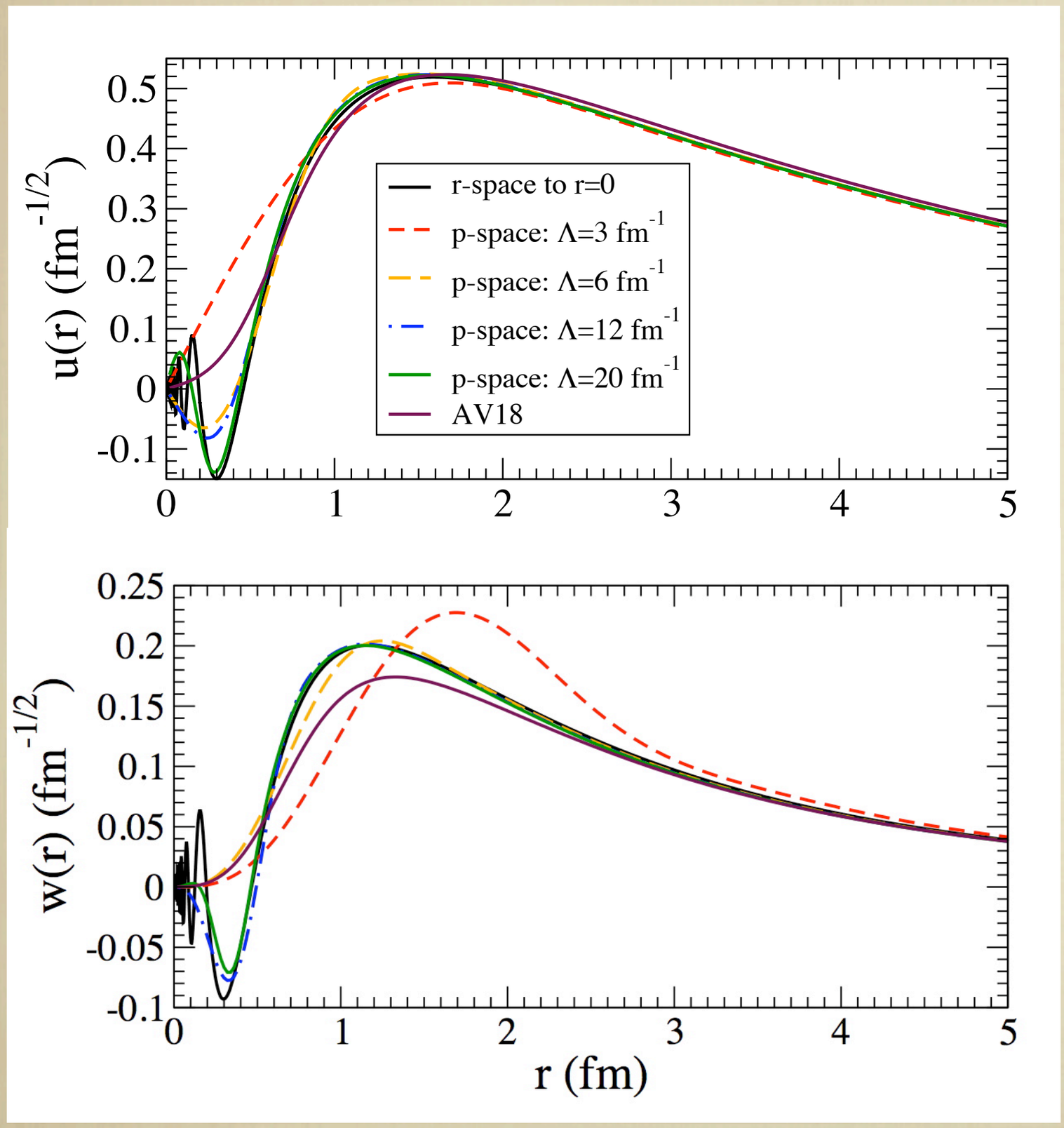

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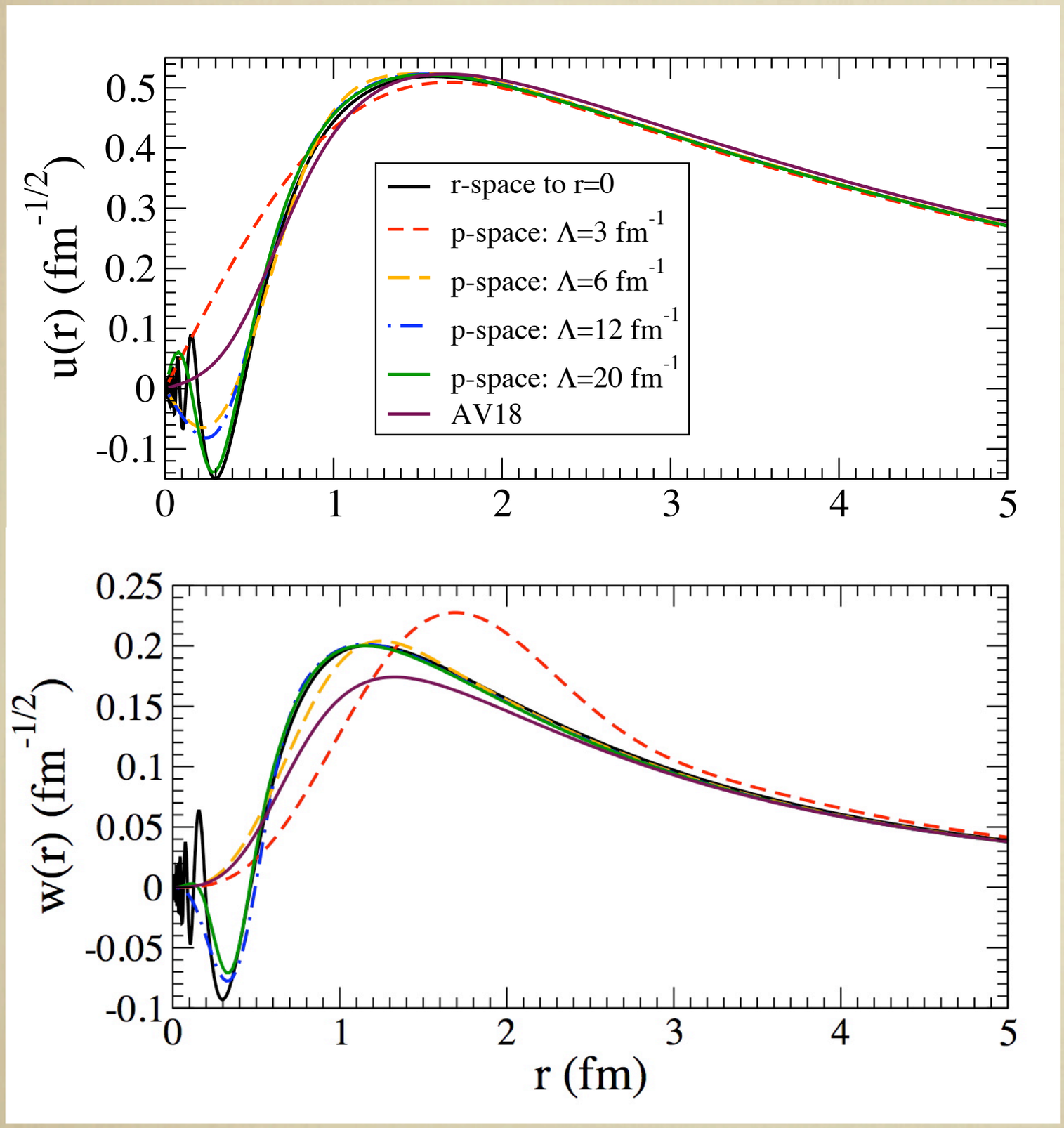

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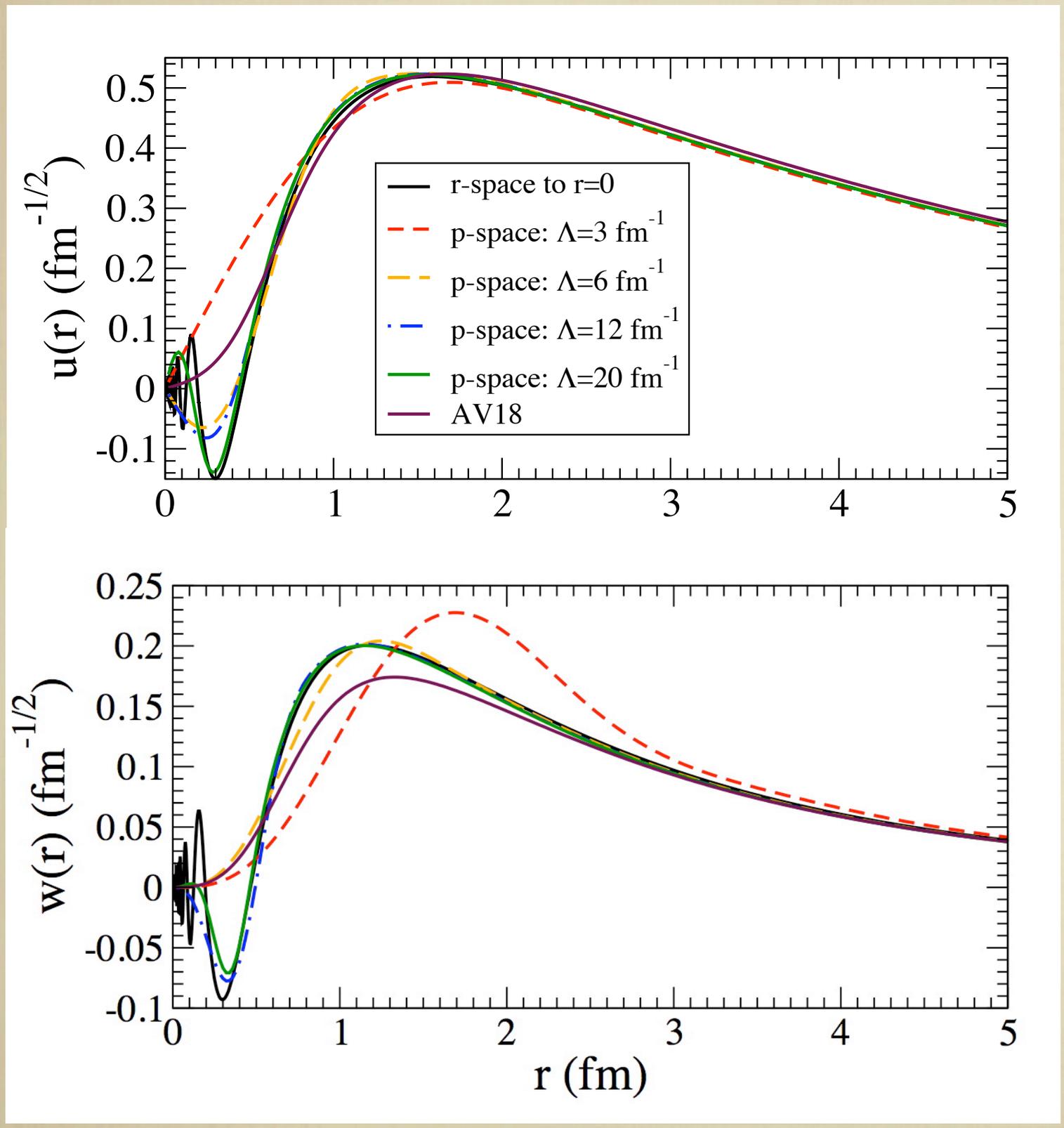

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- Good description of deuteron em form factors for |**q**|<0.6 GeV at leading order

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- Two-pion exchange, additional contact terms at NLO

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)

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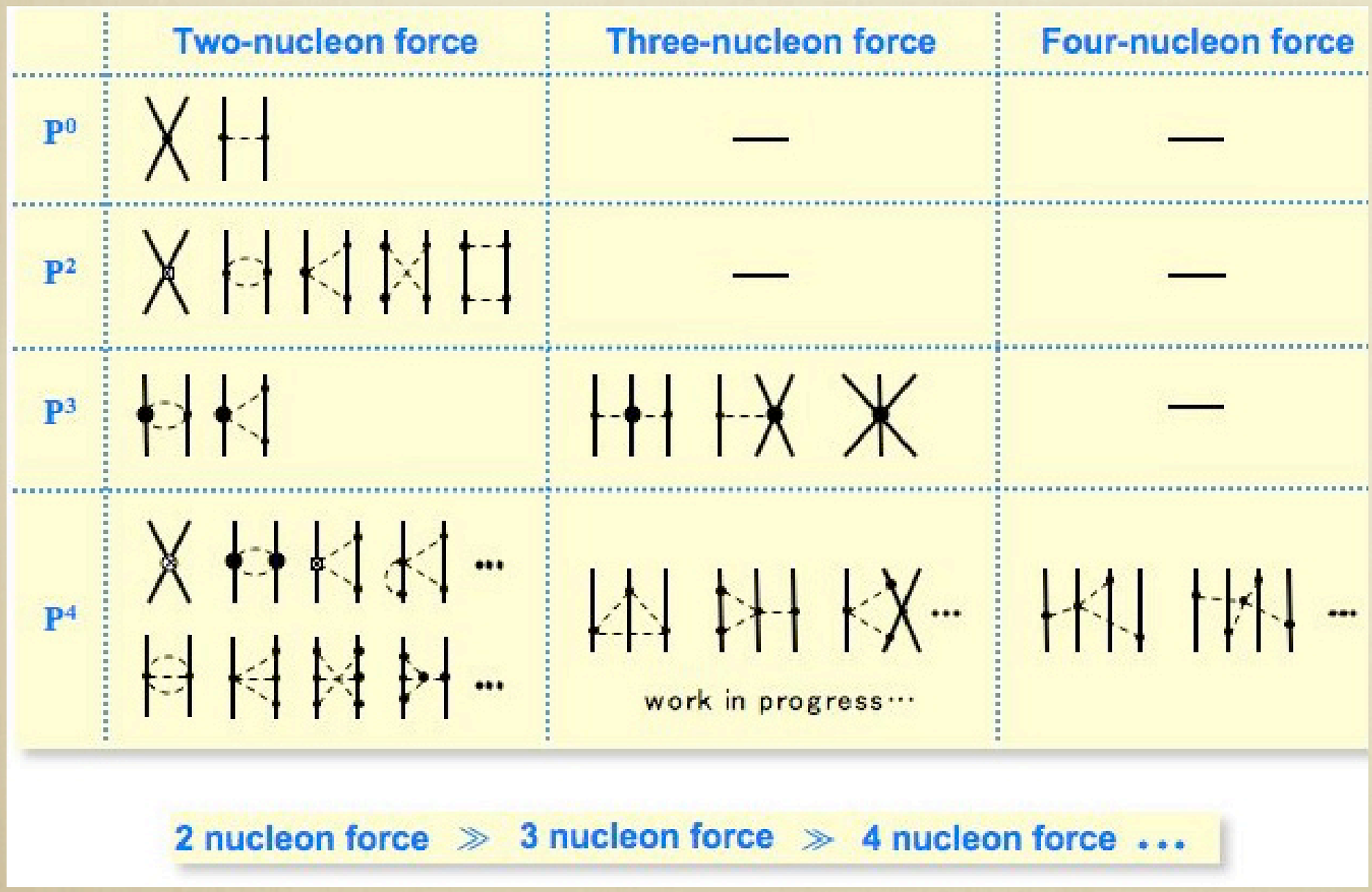
- 3NF at NNLO, free parameters fit in A=3: BE, a_{nd}



Consistent 3NFs, 4NFs

Courtesy
E. Epelbaum

State of the Art: $O(P^4)$

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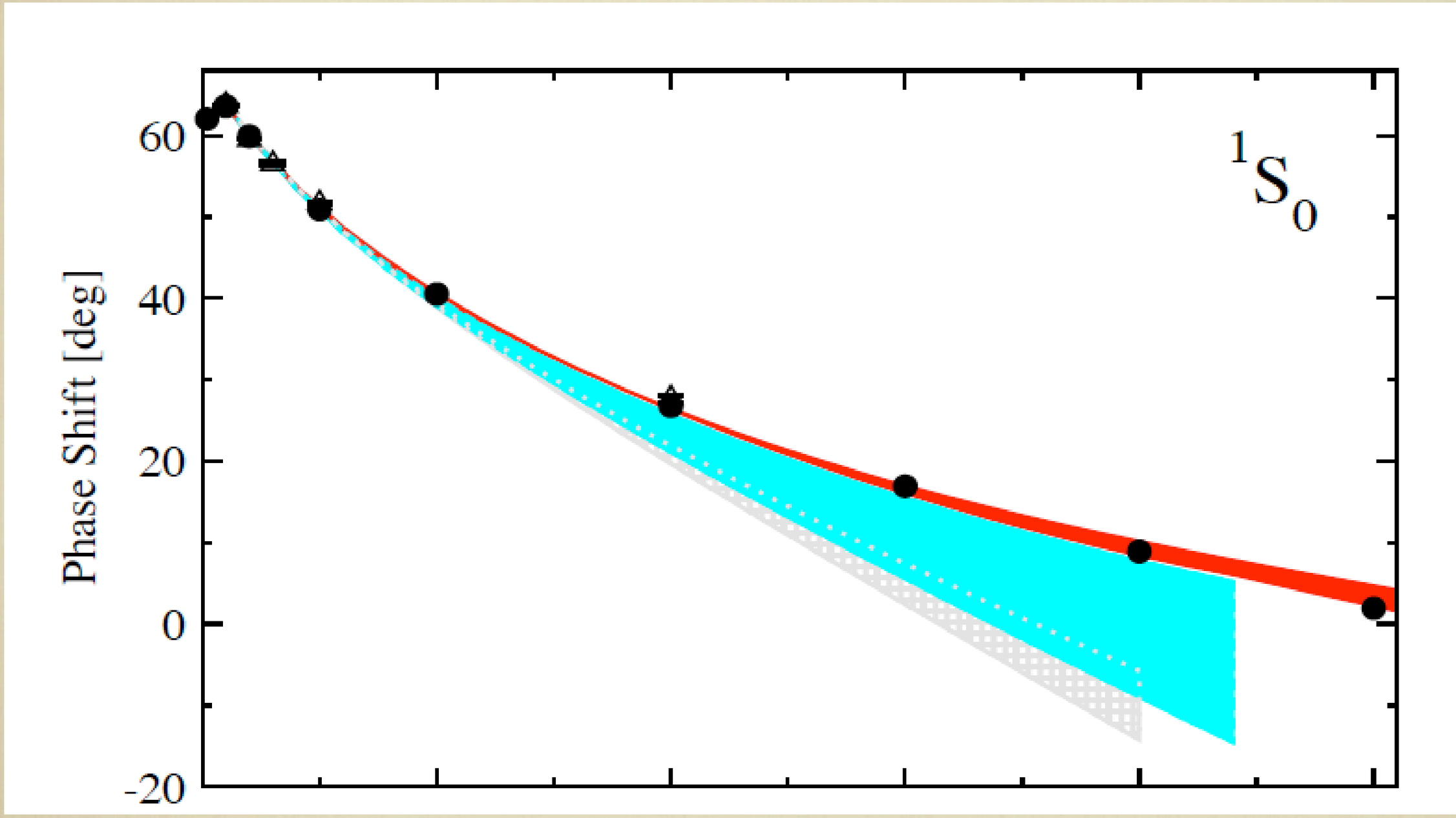

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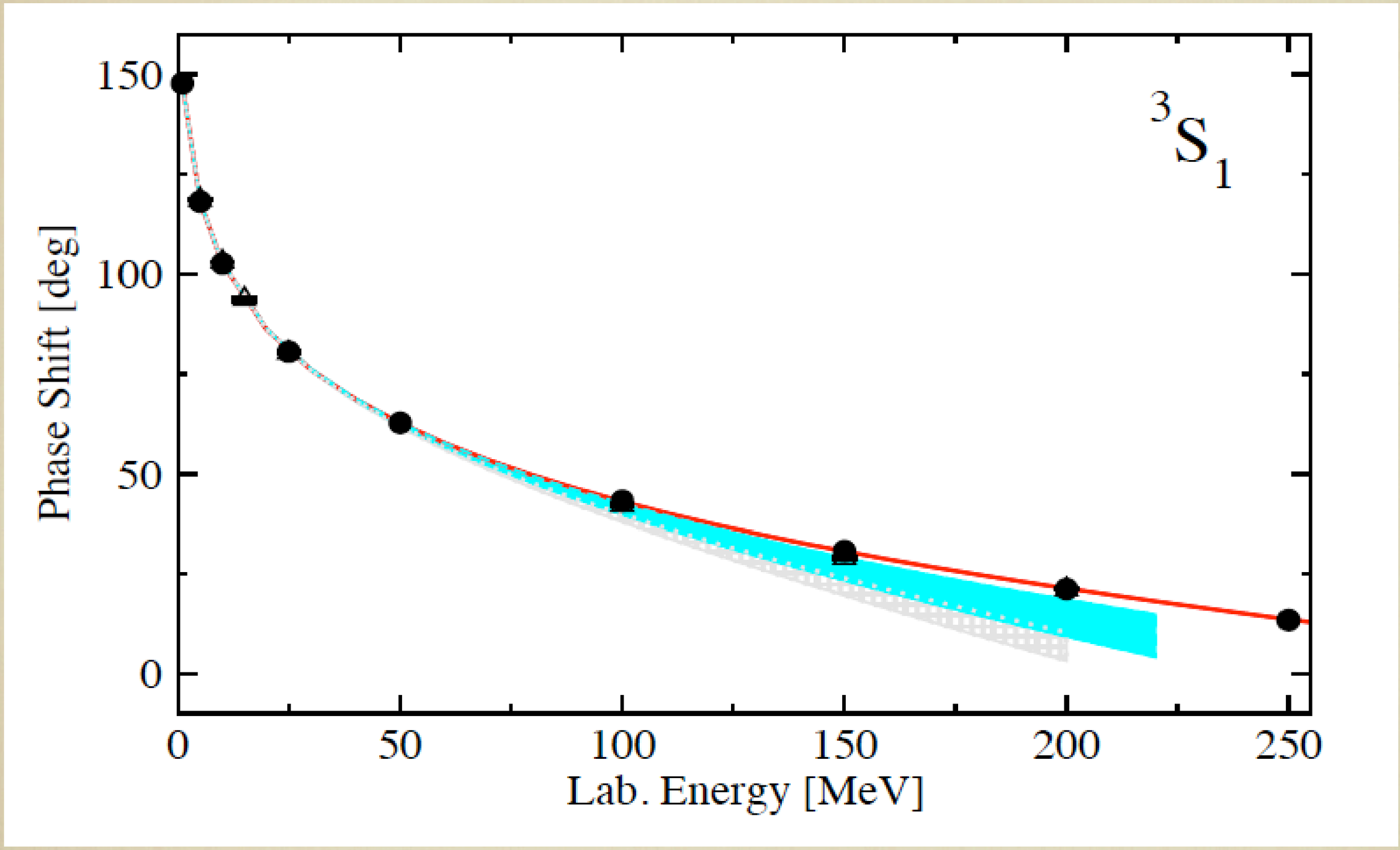

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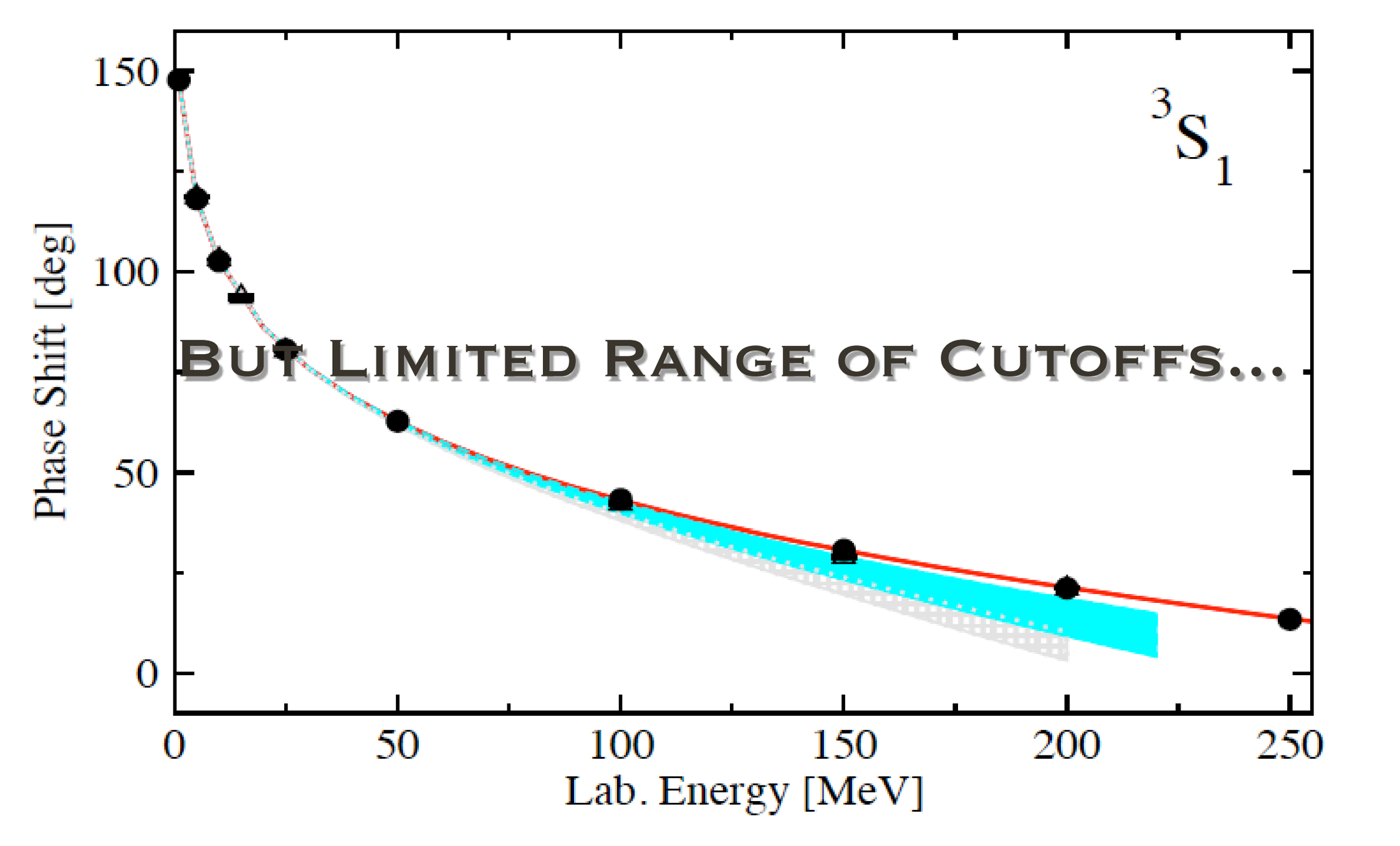

Triplet channel at NNLO

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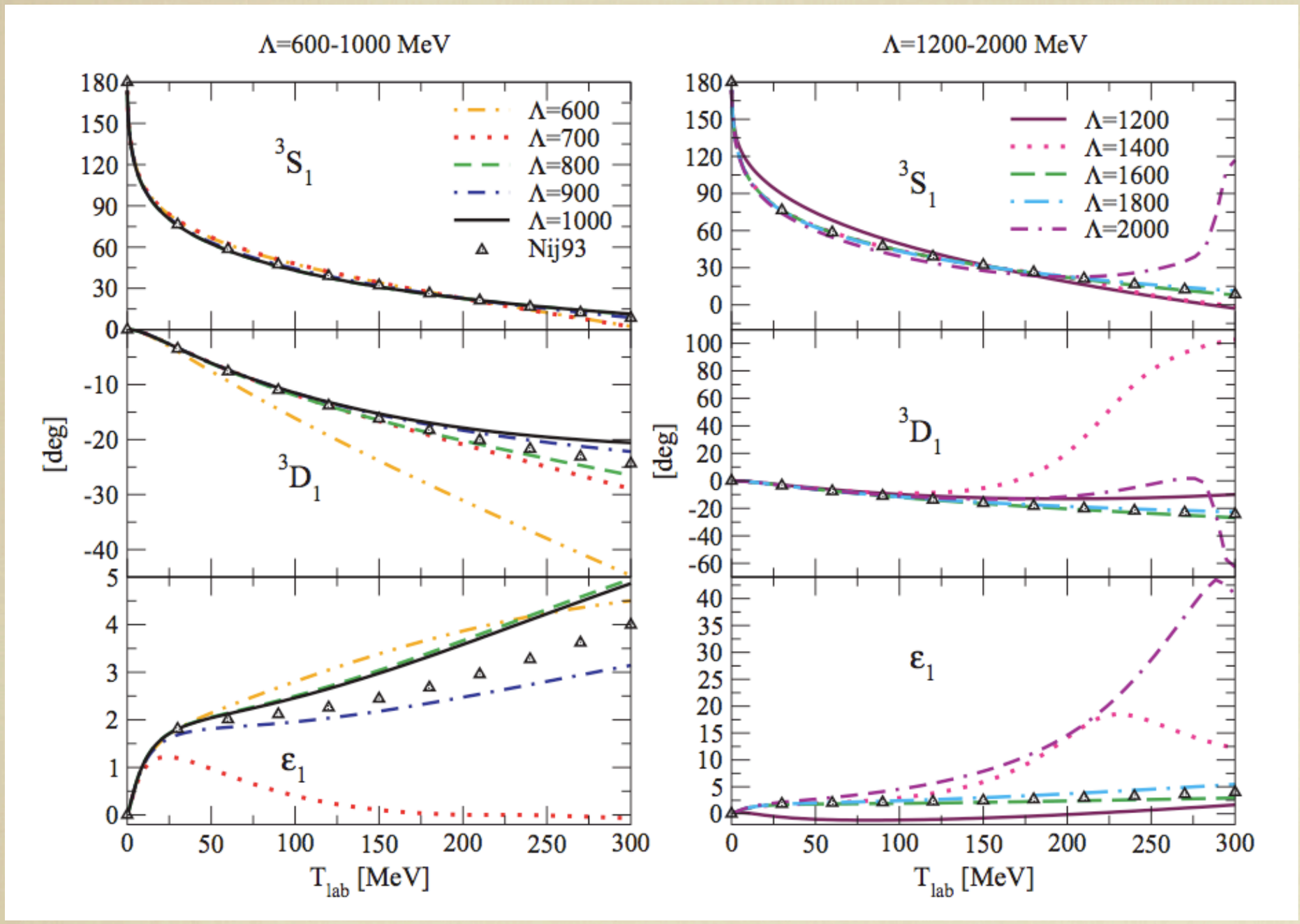

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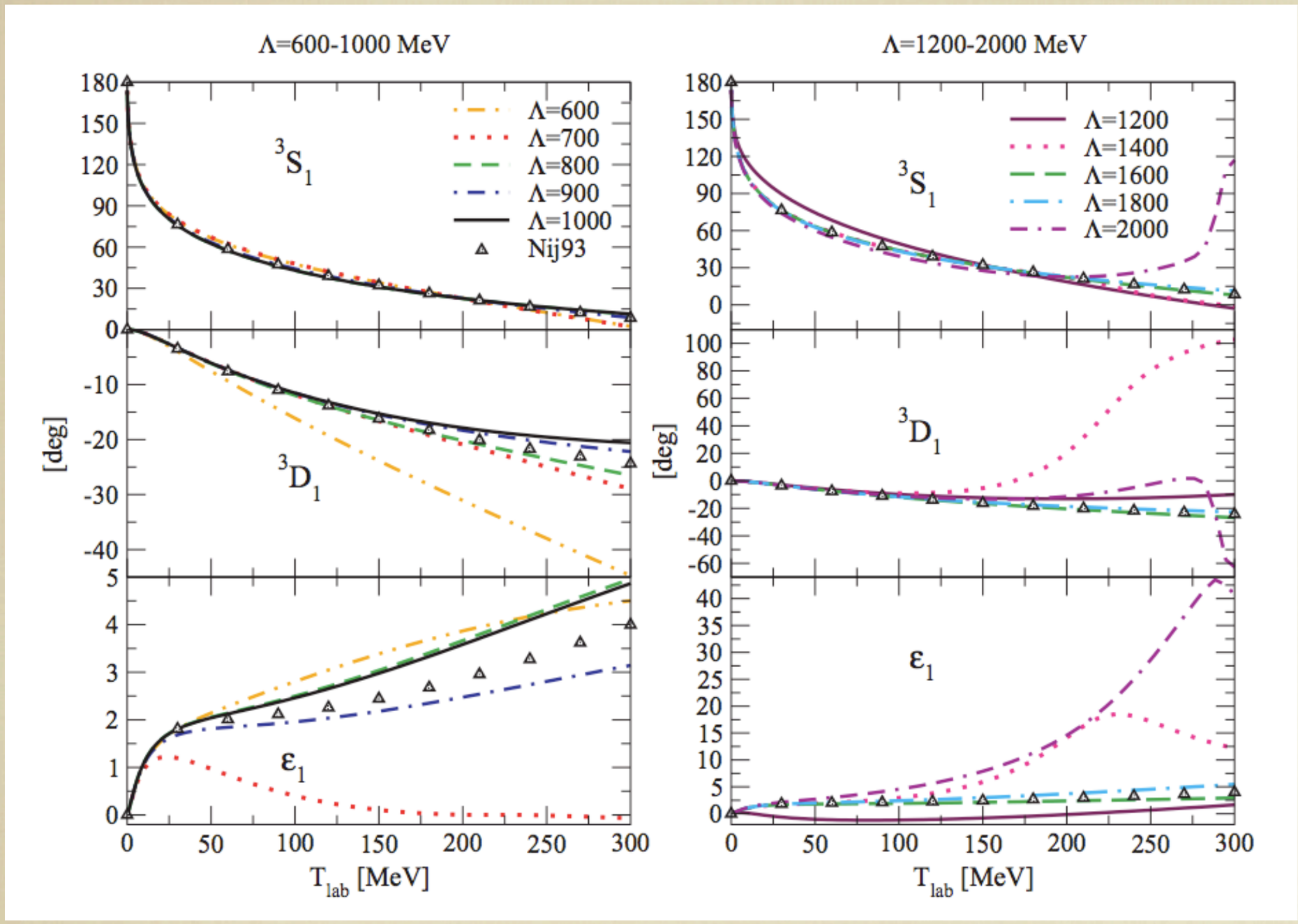

O(P³) [NNLO] χET NN potential: calculated with DR

- Increasingly singular potentials as chiral order increases
- Momentum-dependent short-distance part has limited effect as Λ→∞

Issues for other L

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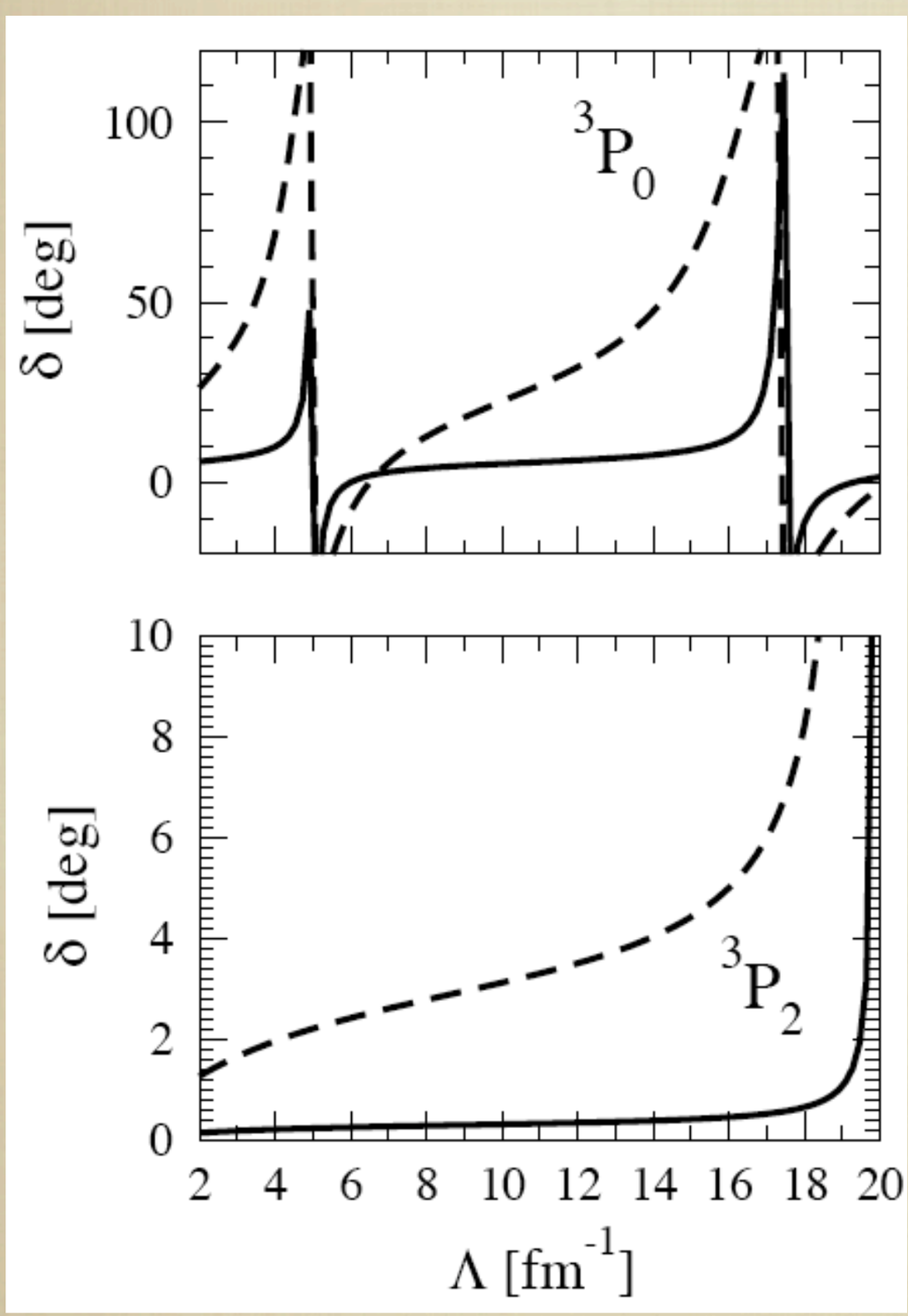
Eiras, Soto (2002); Nogga, Timmermans, van Kolck (2005)



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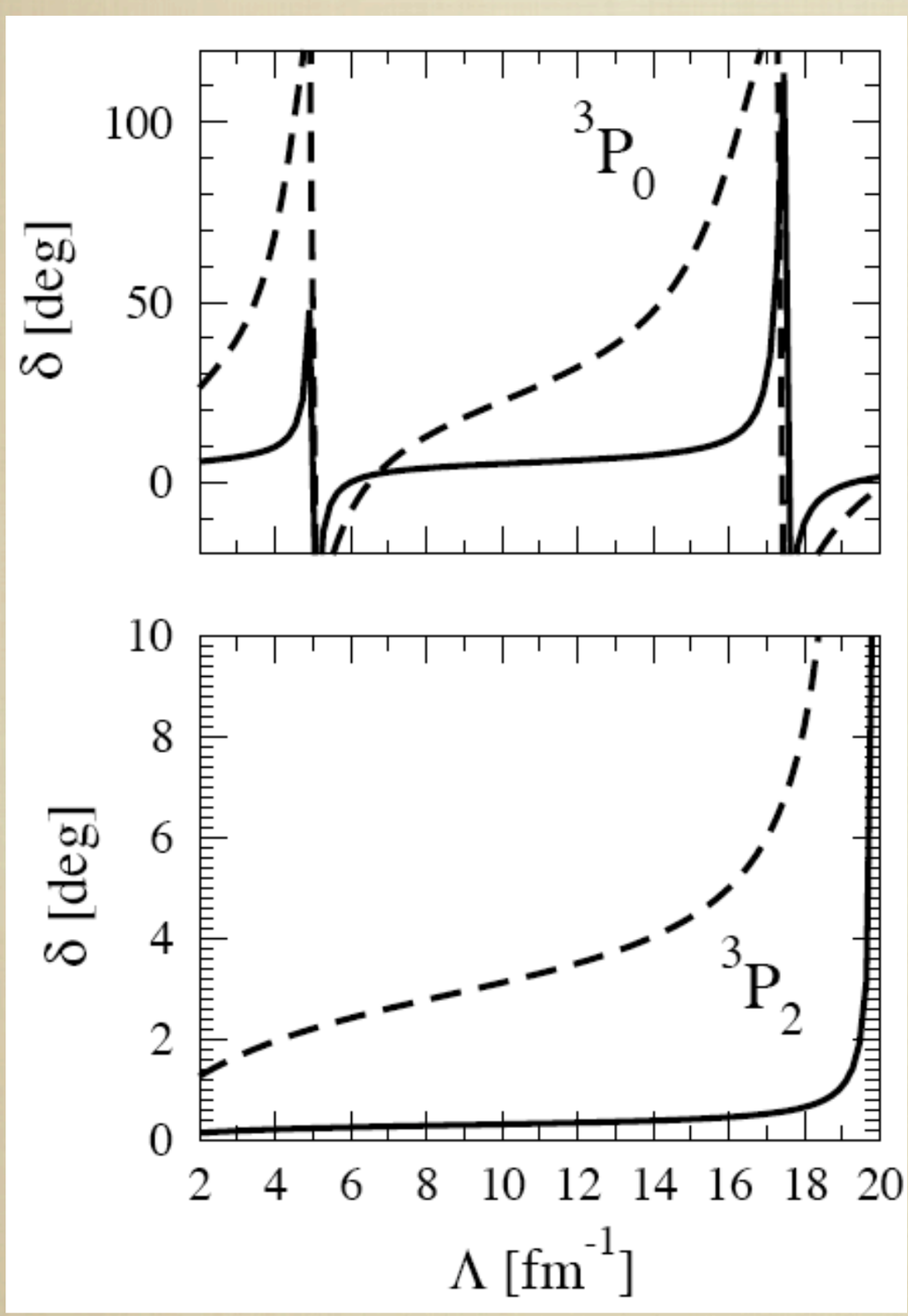

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Pavon Valderrama, Ruiz Arriola

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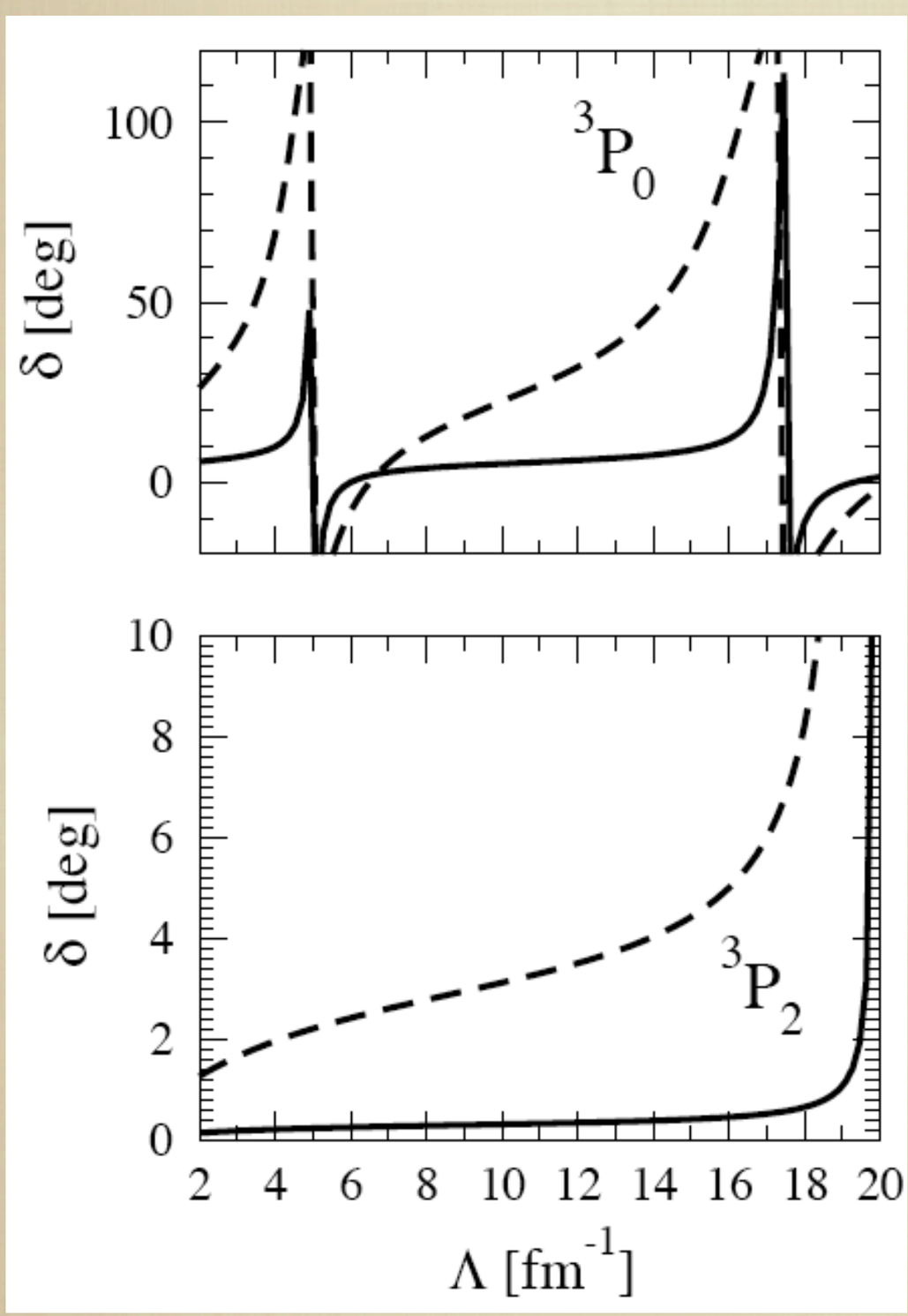

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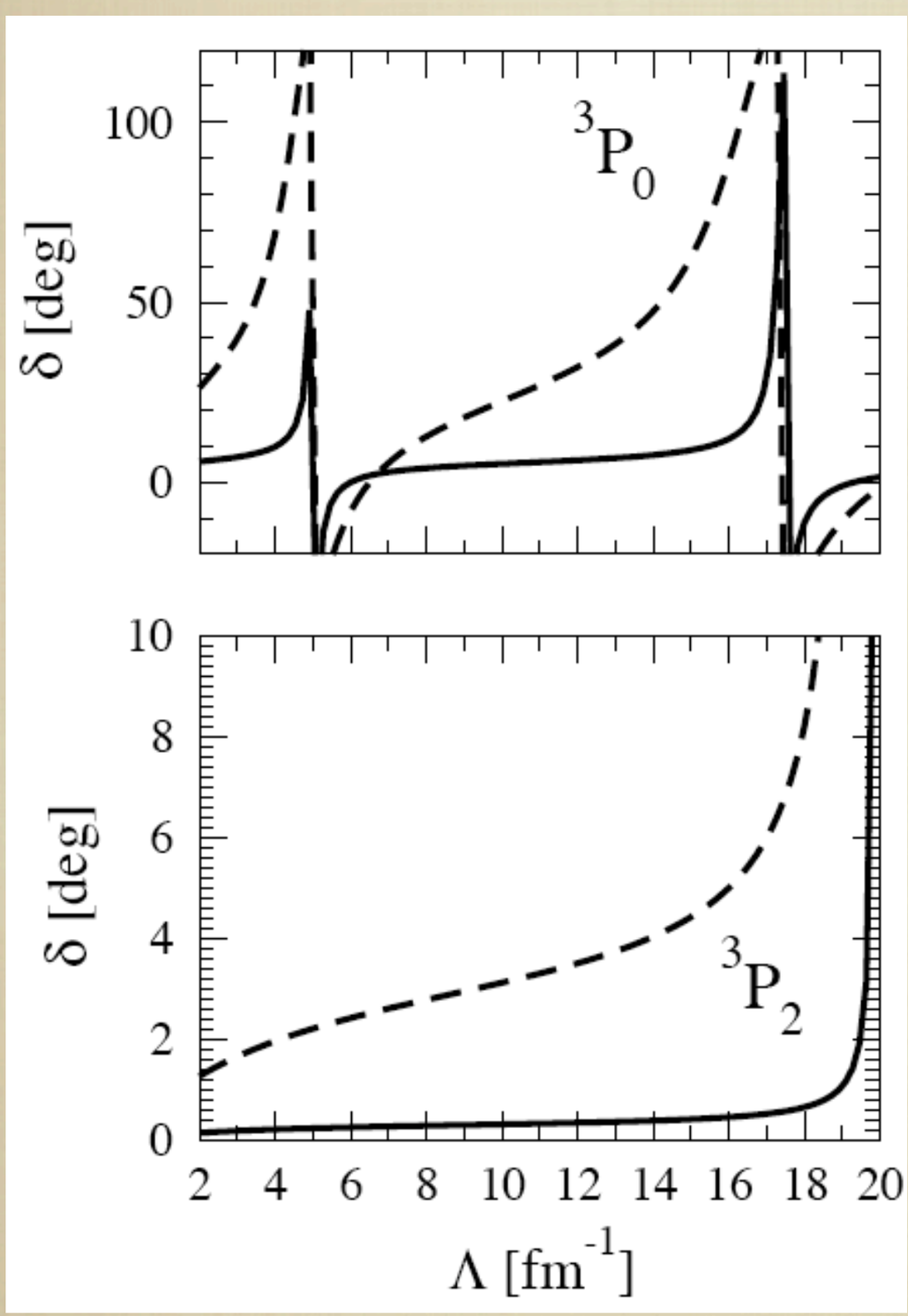

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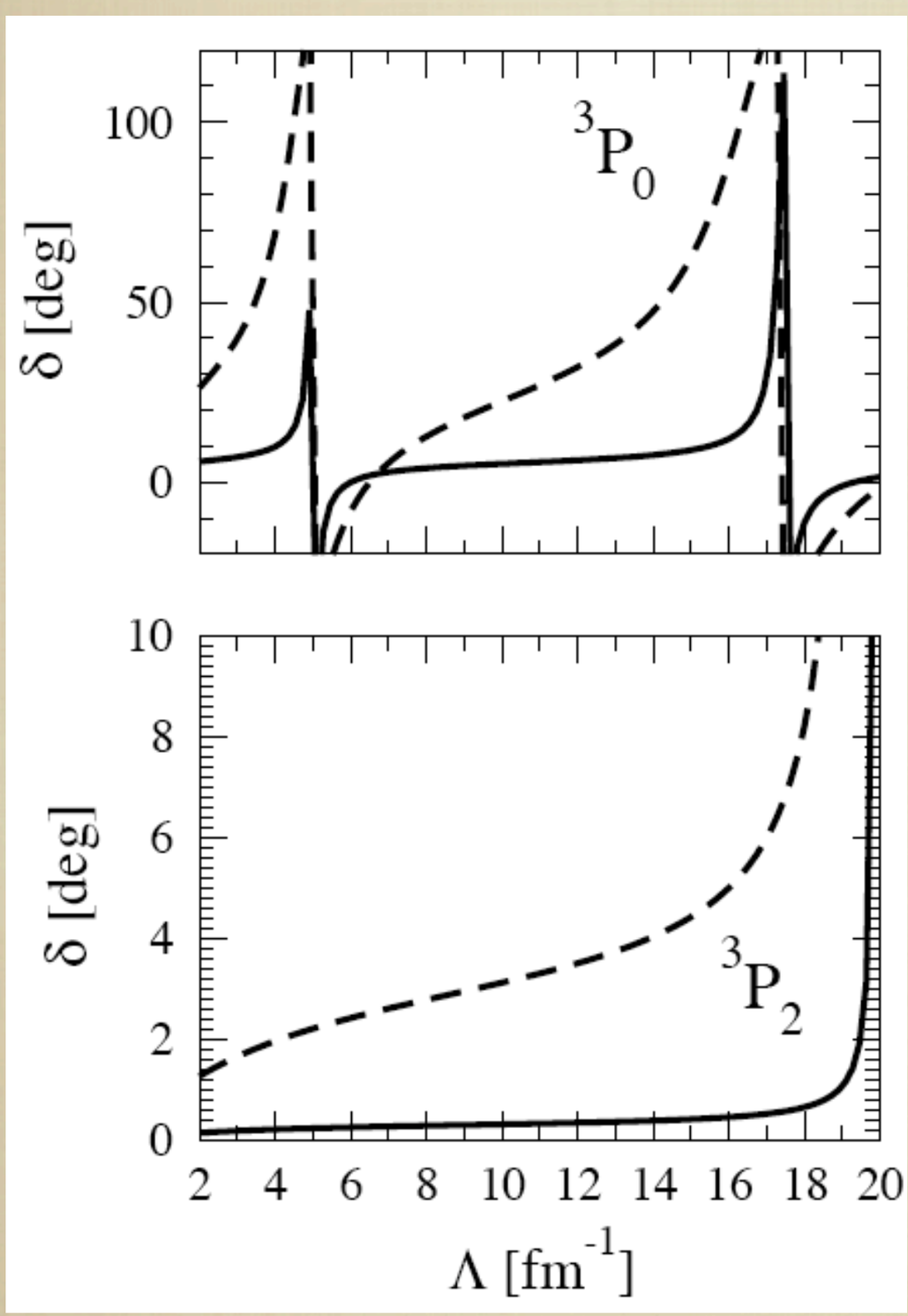

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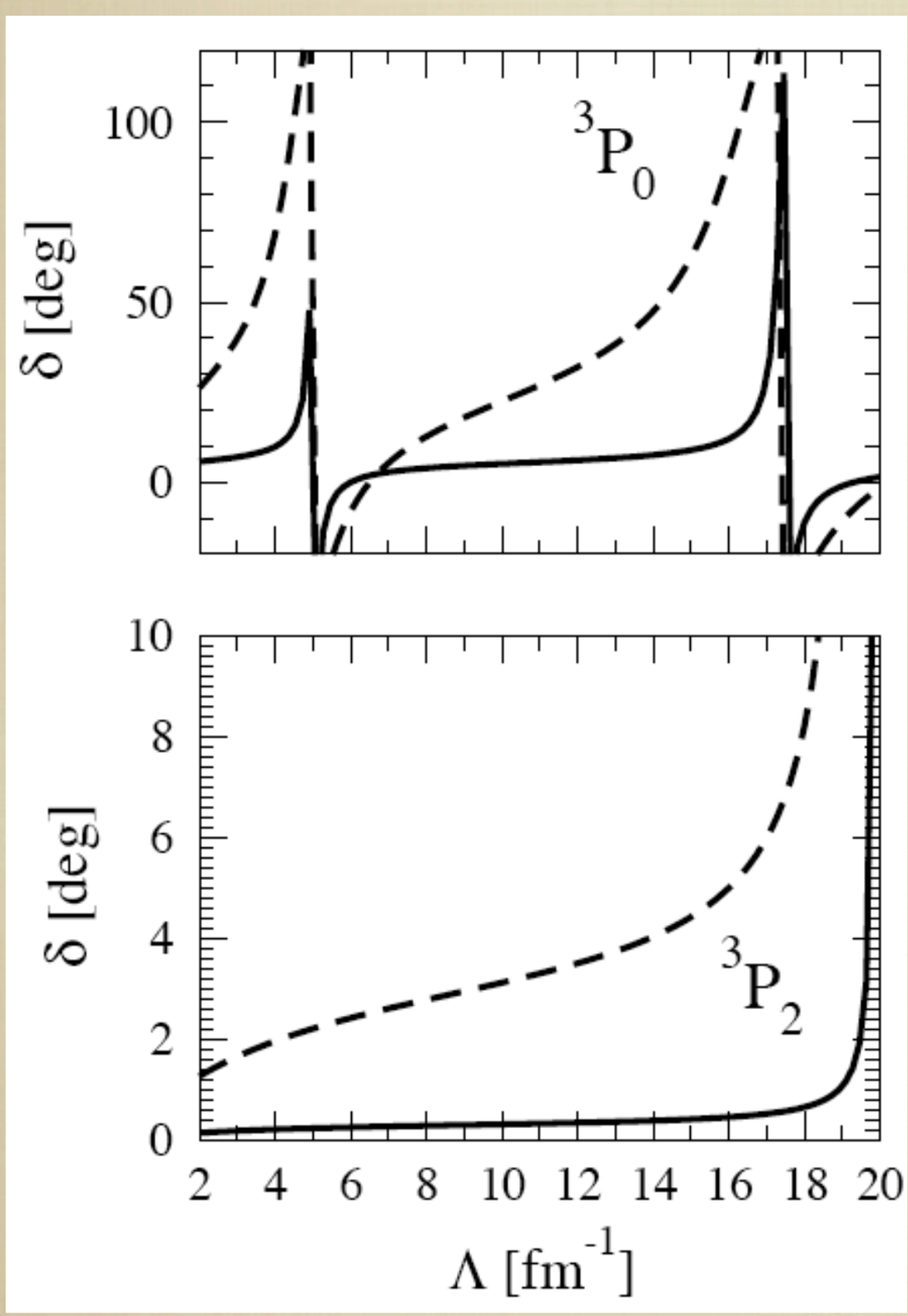

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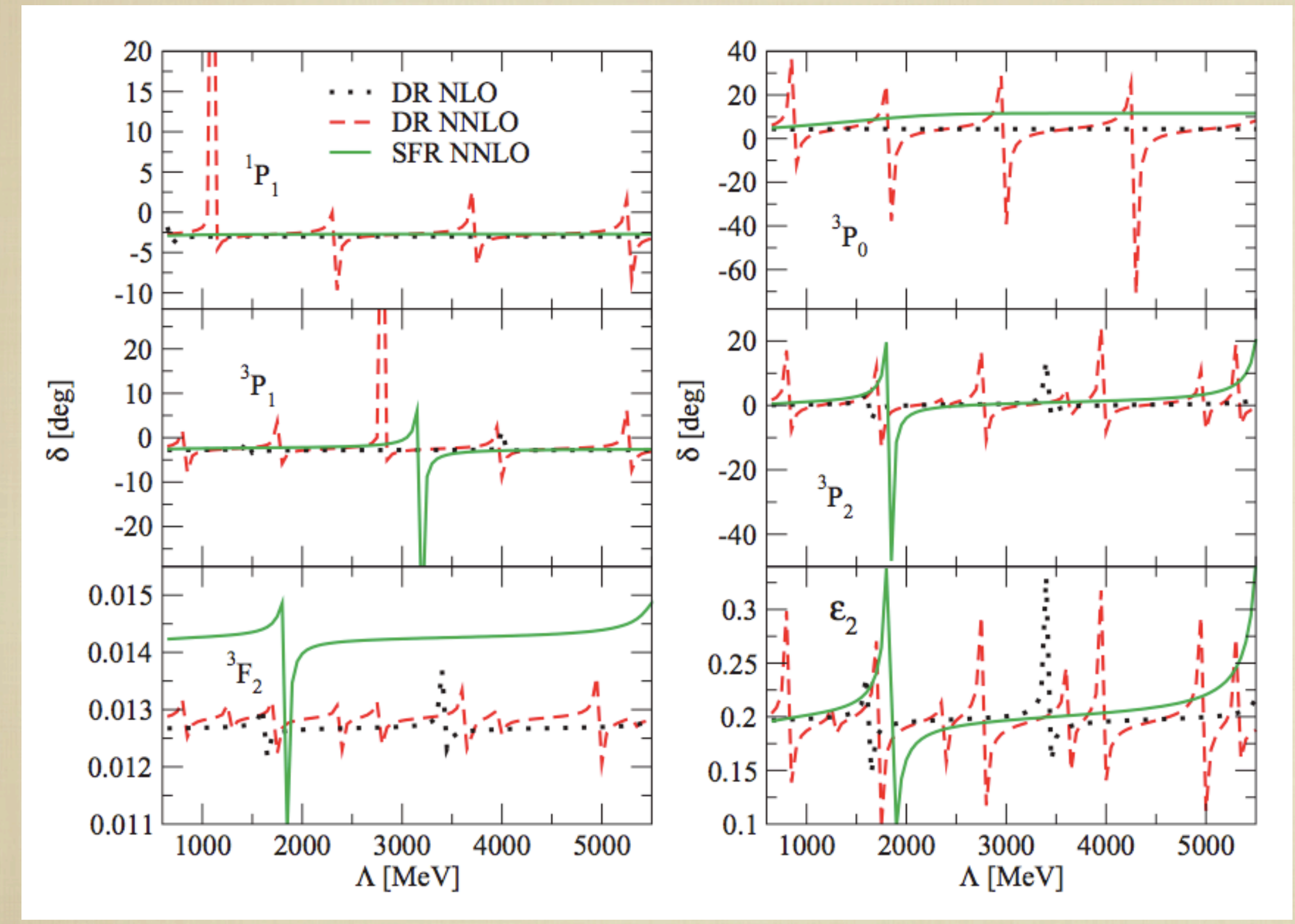
- Ideas not practically implemented

Problems in P waves for large Λ

Yang, Elster, Phillips, PRC 80, 034002 (2009)



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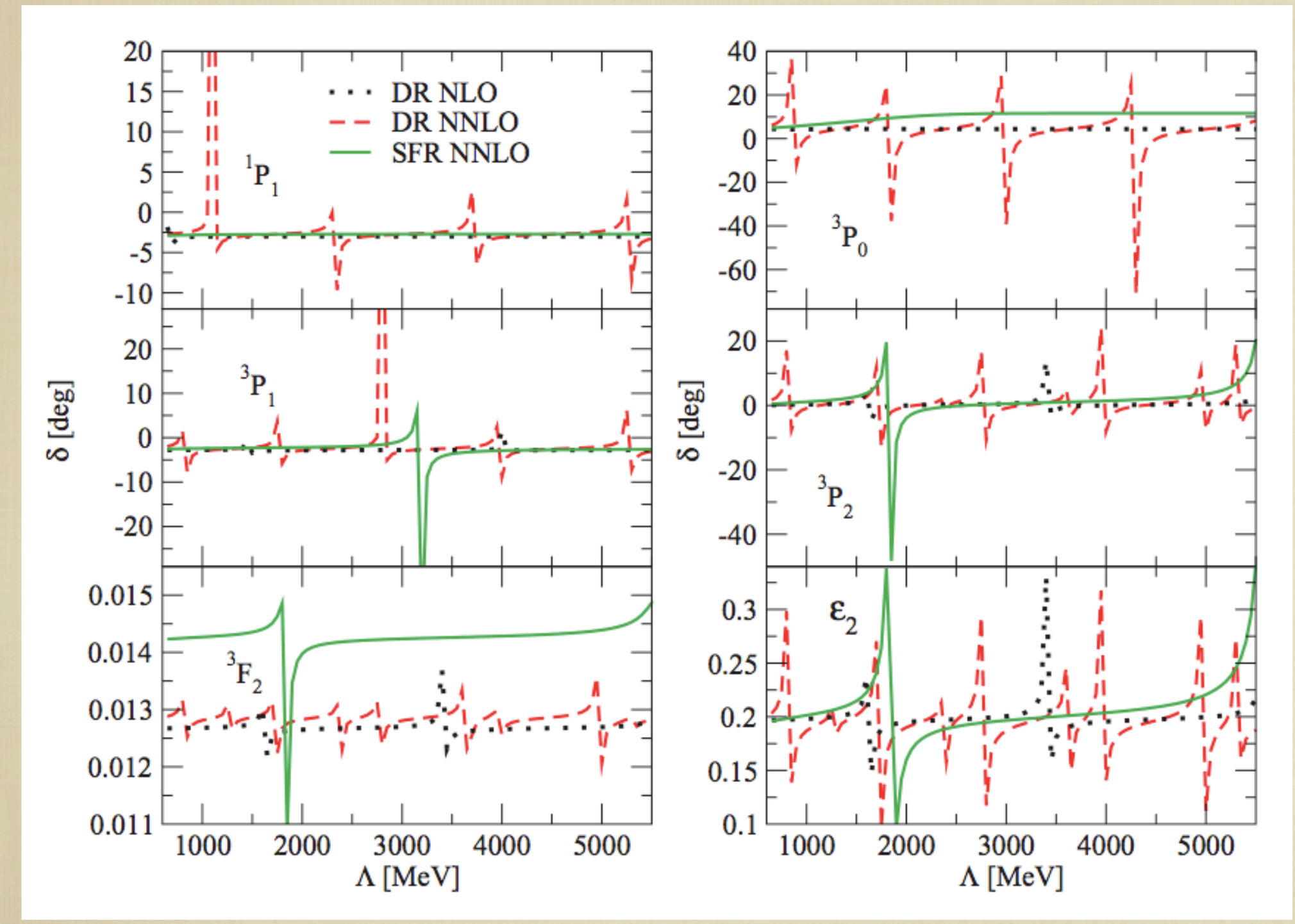

Limited to 0.6 GeV< Λ< 1 GeV

Successes in A=2-4

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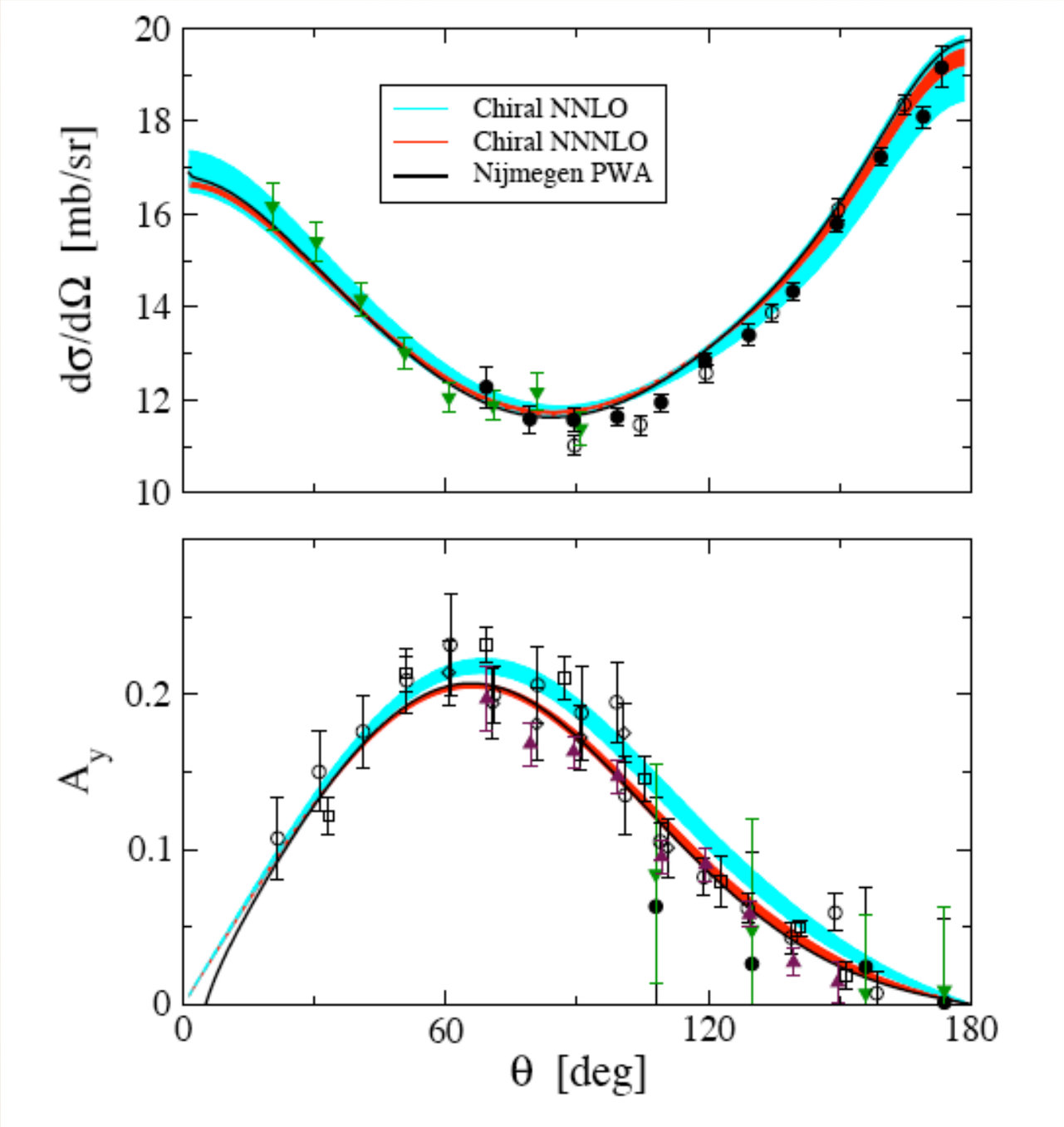

T_{lab}=50 MeV

- N^3LO potential, χ^2/dof good c.f. AV18.

  Entem, Machleidt (2003)

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- Reproduce A=3 and 4 binding energies

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	NLO	NNLO	"Exp."
^{3}H	-7.53..-8.54	-8.68	-8.68
^{4}He	-23.87..-29.57	-29.51..-29.98	-29.6

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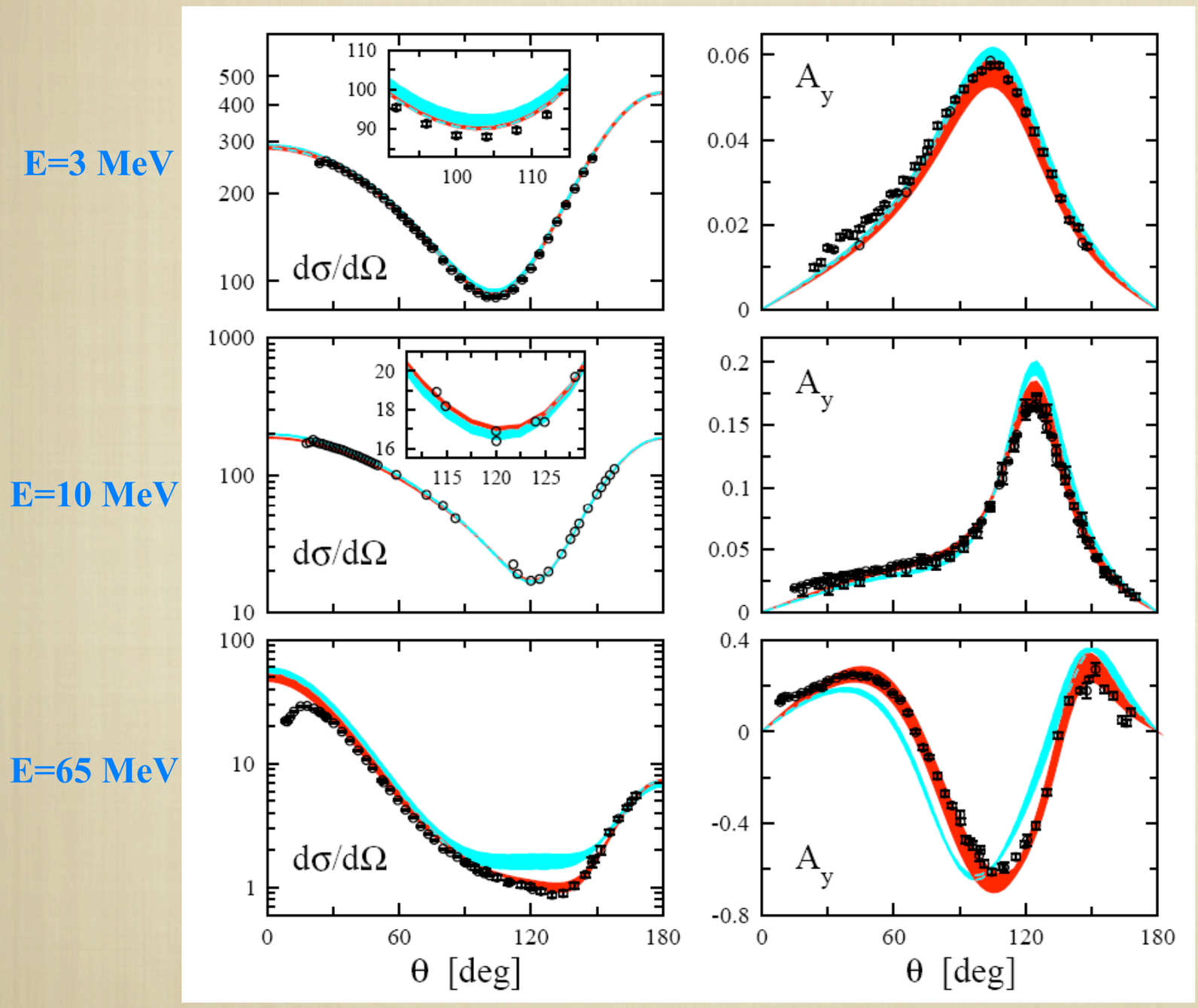

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Testing NN forces in elastic electron-deuteron scattering

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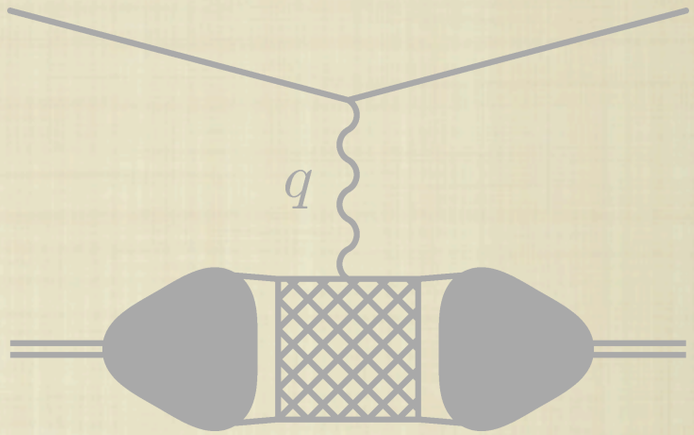
- LO QED: electron couples to J_μ



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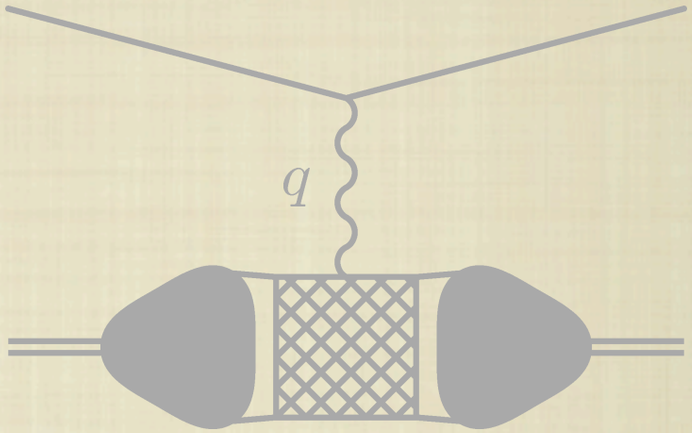

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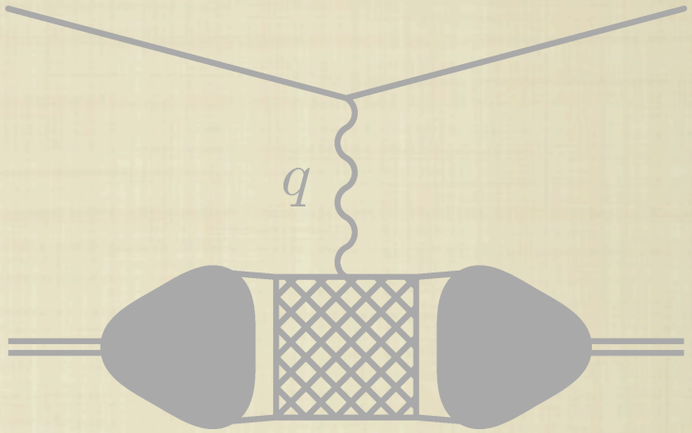

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- Deuteron form factor: $G_C(|\mathbf{q}|) = \int dr\, j_0\left(\frac{|\mathbf{q}|r}{2}\right)[u^2(r) + w^2(r)]$
- Change Q^2=-q^2⇒change spatial resolution

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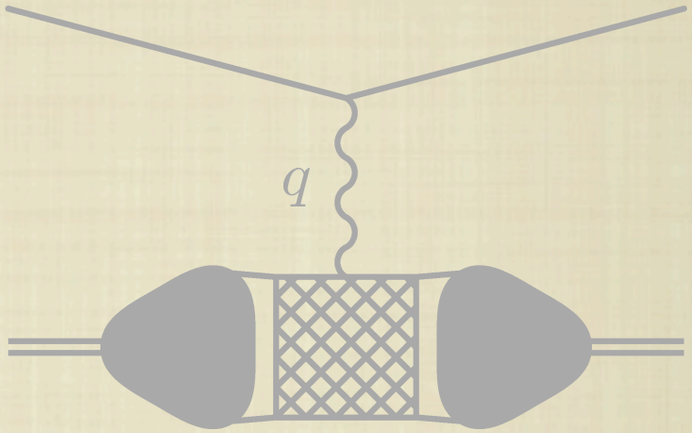

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- **Prediction** of QED and NN force model

Predicting A(Q)

$$A(Q) = G_C^2(Q) + \frac{8}{9}\eta^2 G_Q^2(Q) + \frac{2}{3}\eta G_M^2(Q); \quad \eta = \frac{Q^2}{4M_d^2}$$

- JLab Experiment in Hall A
- 2-3% precision for 0.2<|**q**|< 0.8 GeV/c
- Goal: resolve discrepancy between Mainz and Saclay data
- Here: 0.2<|**q**|<0.5 GeV/c⇒η=1.6%, so A↔G_C
- We will compute G_C to O(eP^4), estimate uncertainties

Kaiser, Park, DP (2010)

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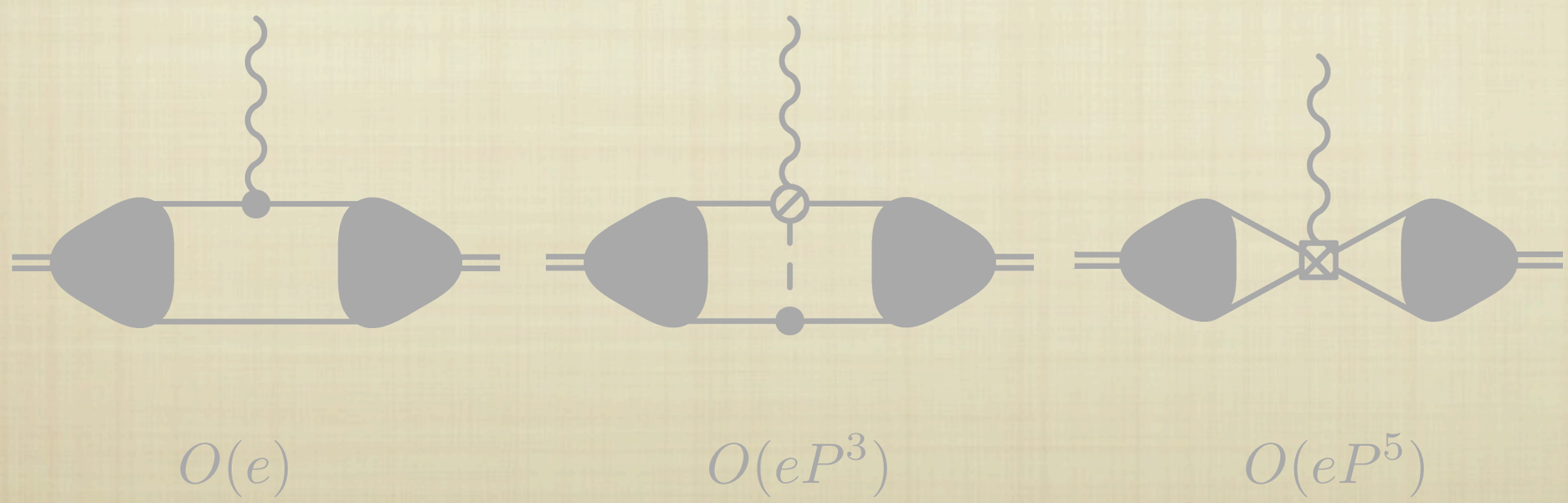

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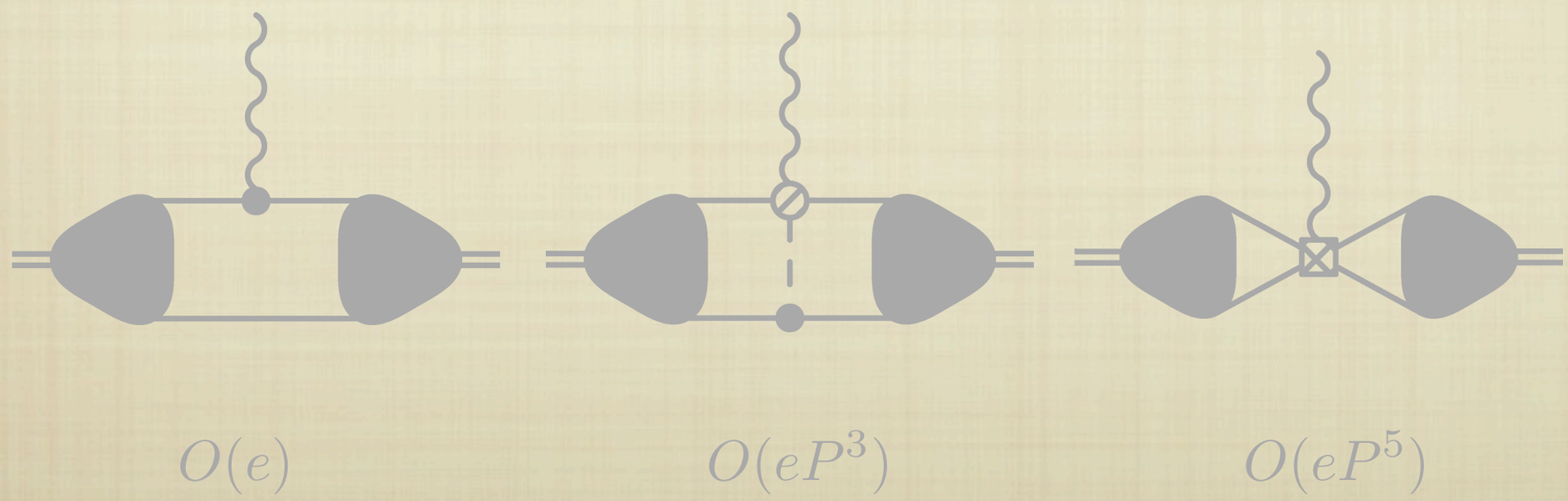

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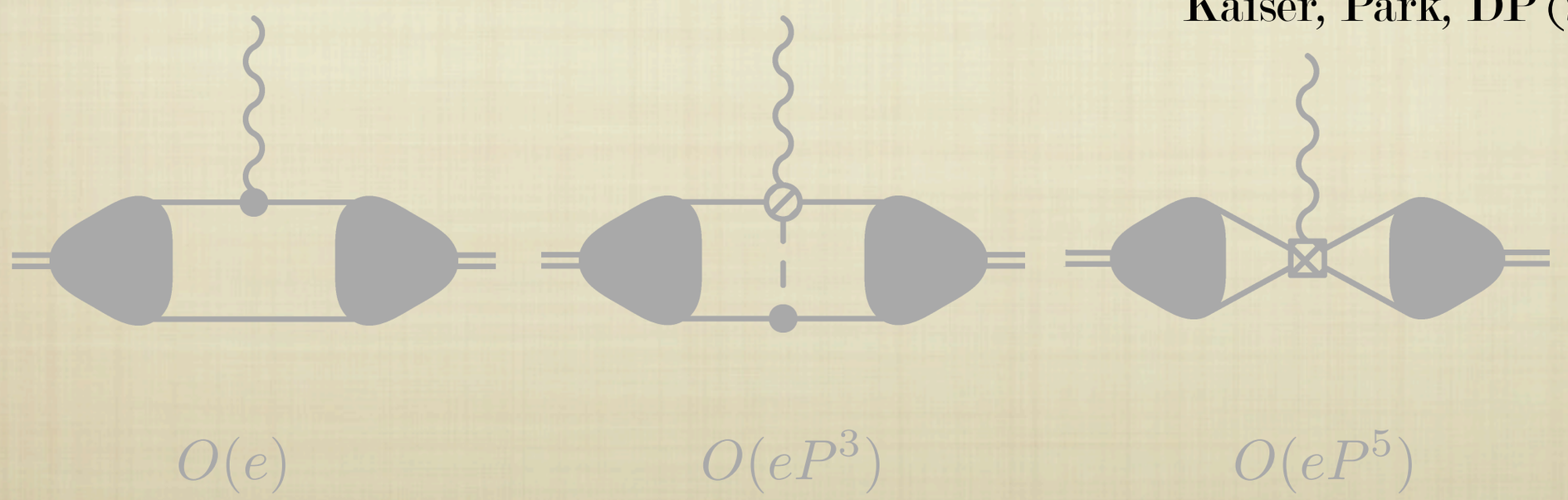
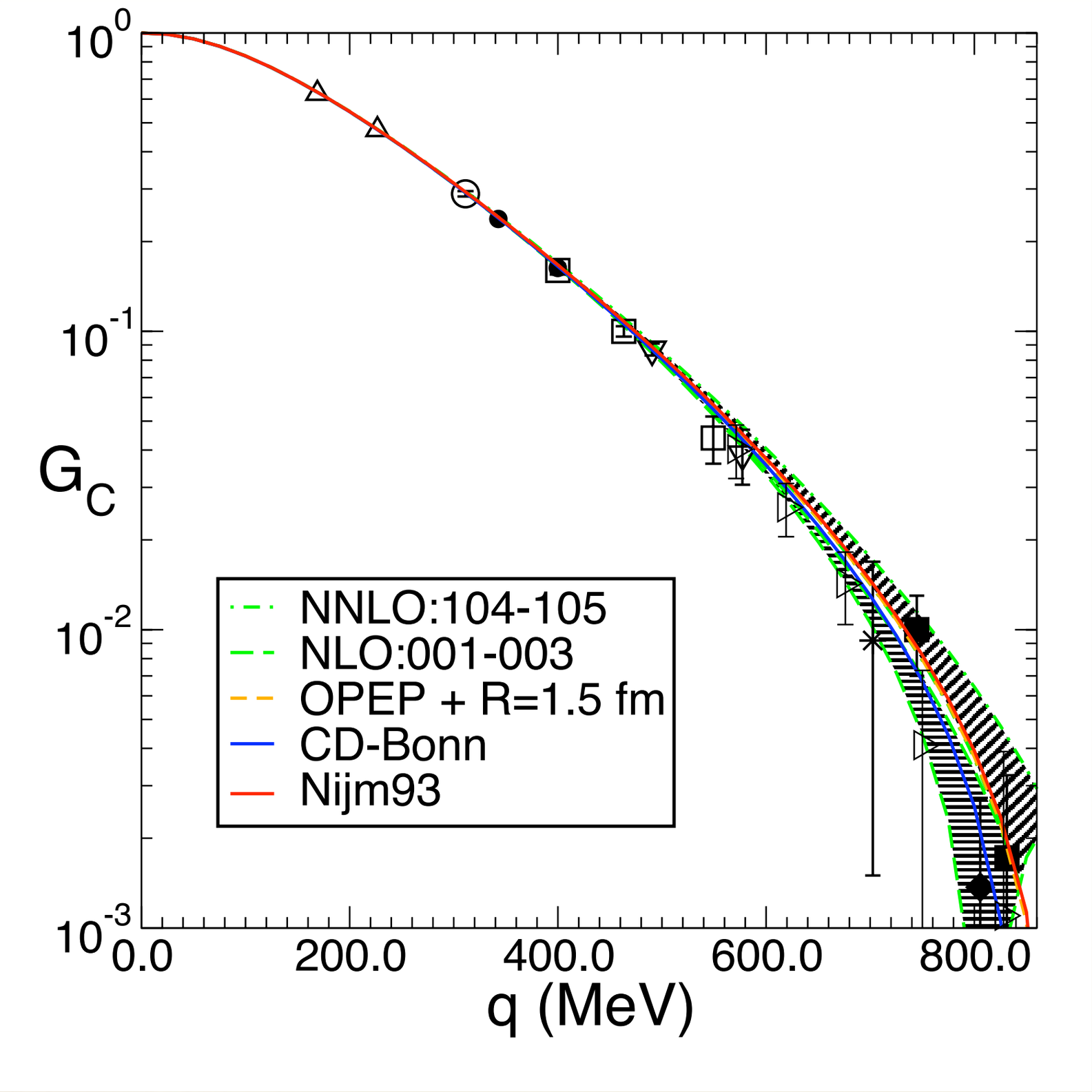
Vanishes!

Kölling et al. (2009)

Kaiser, Park, DP (2010)



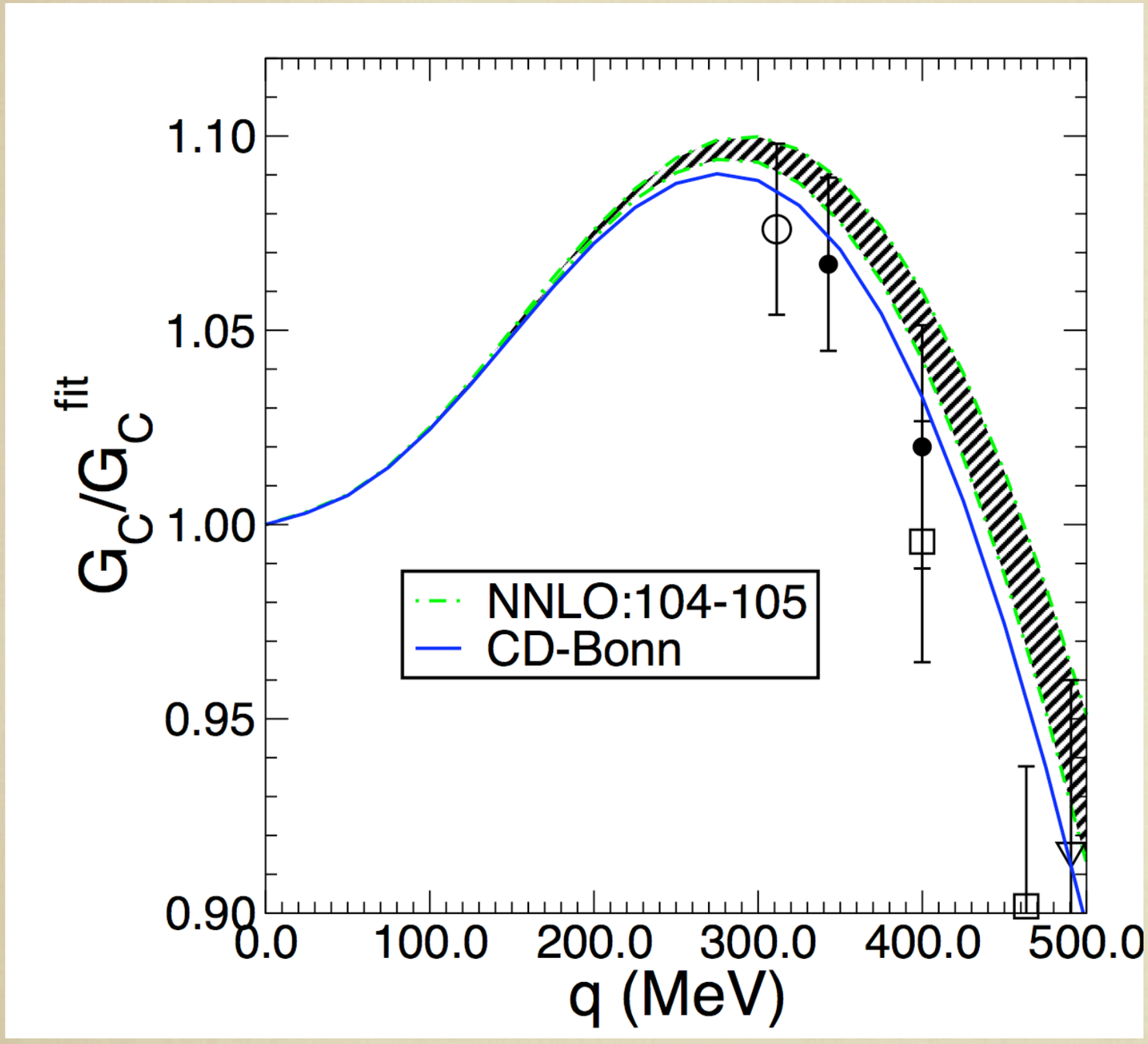
χET for G_C to NNLO, $O(eP^3)$



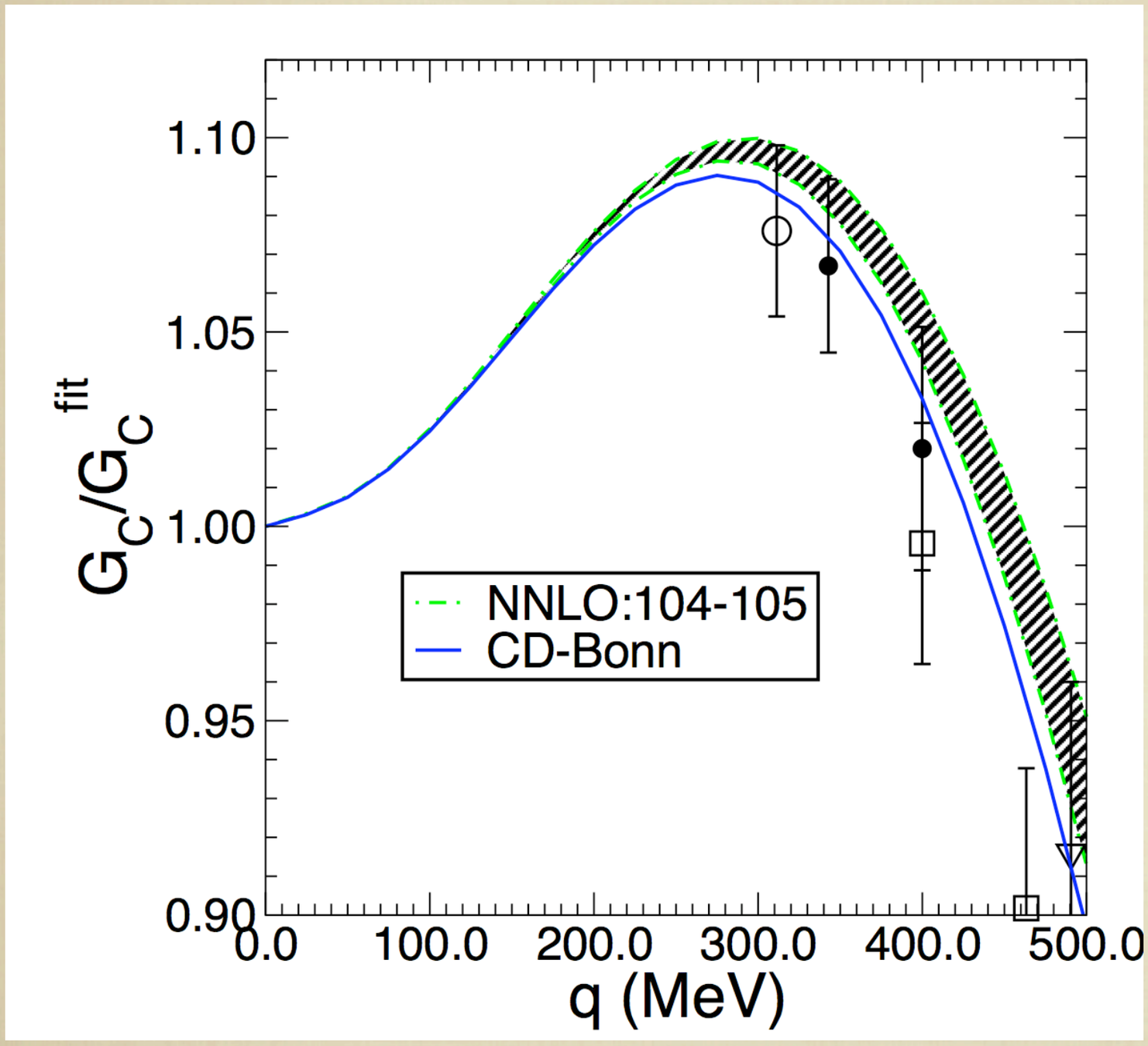
- Good J_0 convergence
- G_C insensitive to $r \sim 1/\Lambda$ physics
- G_M more sensitive to $r \sim 1/\Lambda$; only NLO calculation exists

DP, J. Phys. G 34, 365 (2007)

G_C/G_C^{fit}


1.10
1.05
1.00
0.95
0.90
0.0
100.0
200.0
300.0
400.0
500.0
q (MeV)
G_C/G_C^{fit}
NNLO:104-105
CD-Bonn

G_C/G_C^{fit}



- More data: only some Abbott et al. data here
- Role of nucleon ffs; here Mergell et al.

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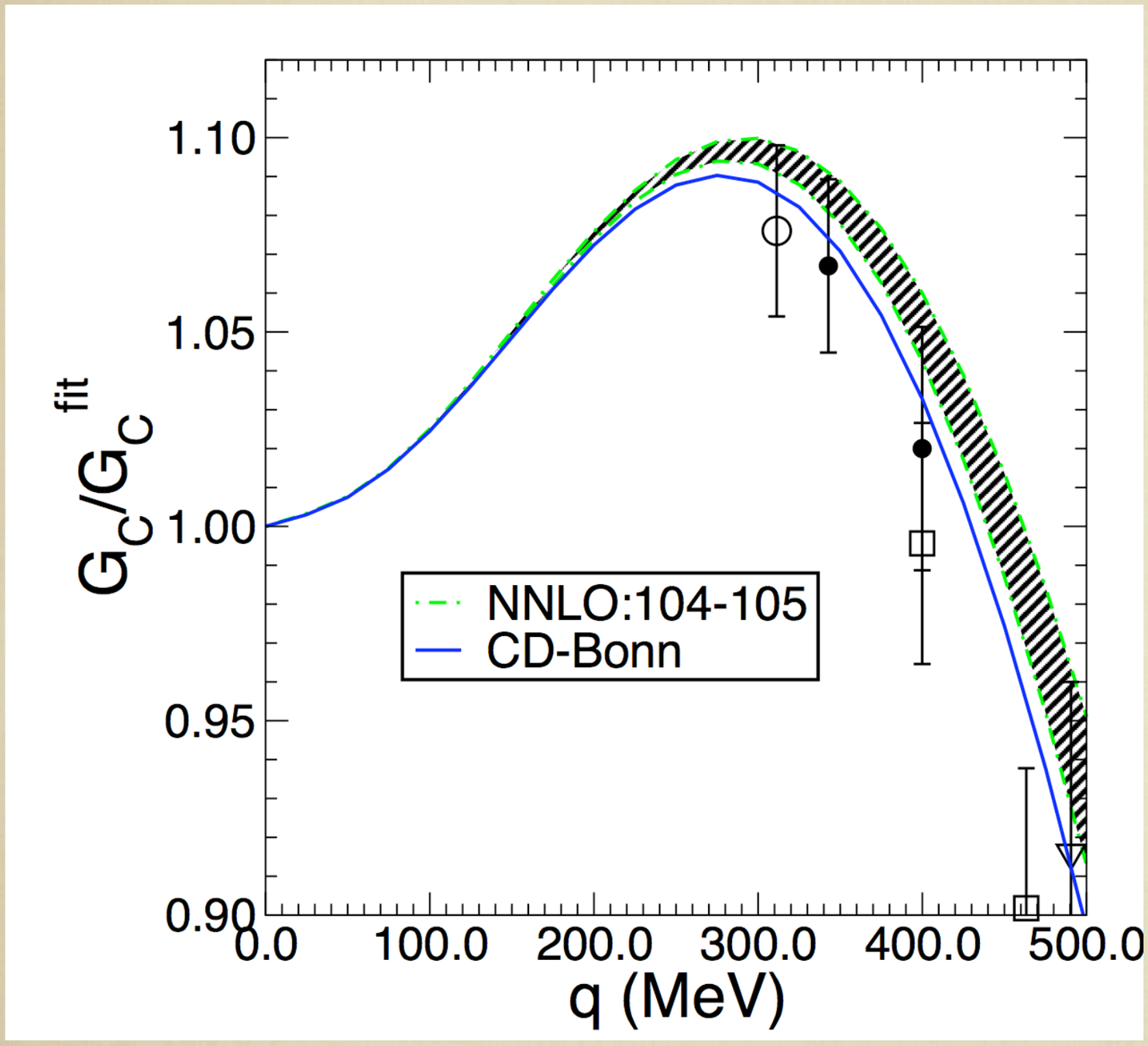

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- Hydrogen level shift: $\langle r_{pt}^2\rangle^{1/2}$=1.9753(10)

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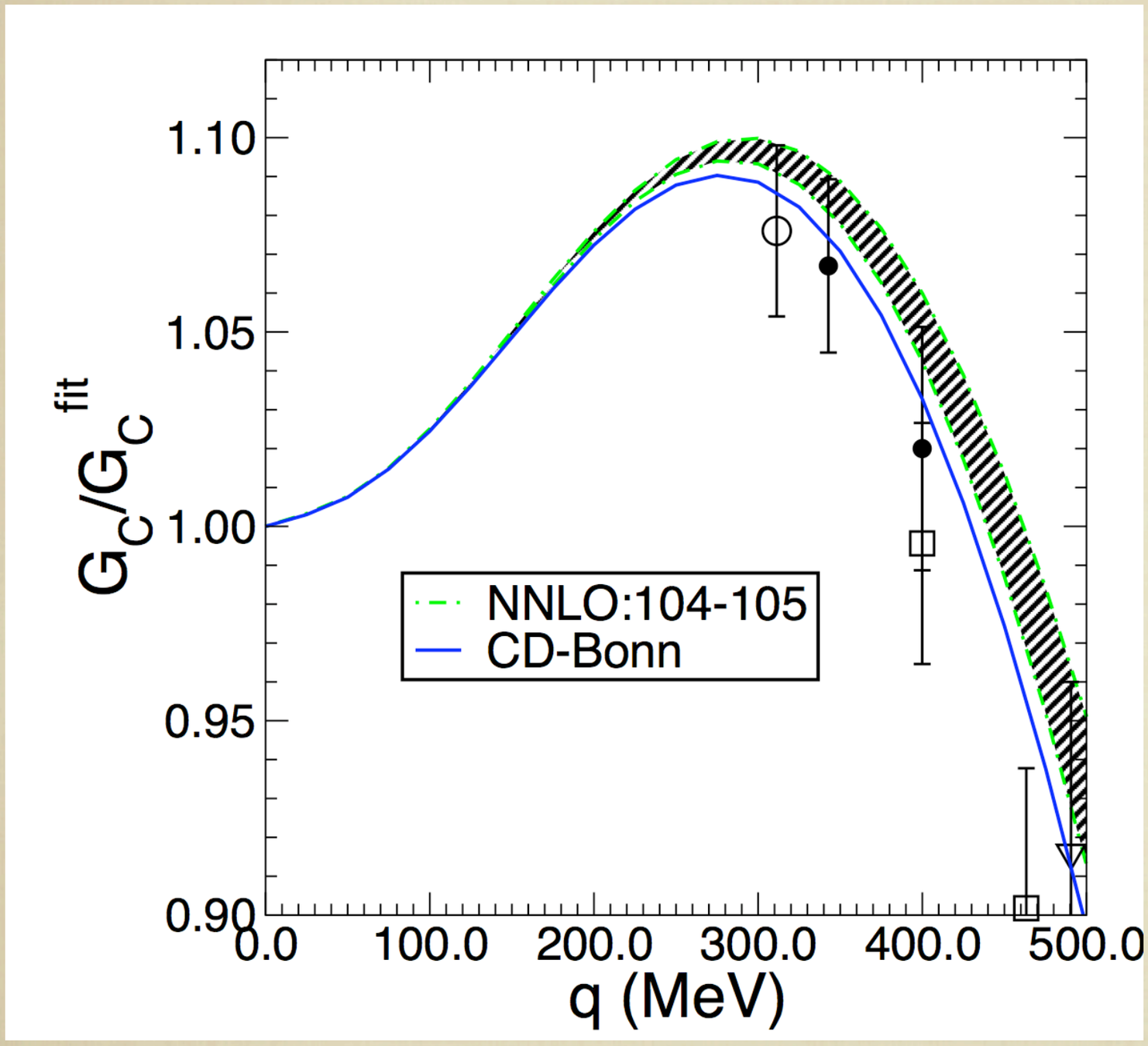

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- At most 0.6% shift in ratio at Q=0.5 GeV/c

Conclusion

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see also Pastore et al., PRC 80, 034004 (2009)

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- Magnetic moments of light nuclei, plus capture, e.g. nd→tγ, n^{3}He→^{4}Heγ. Hierarchy of mechanisms there not necessarily in accord with counting a la Weinberg.

  Pastore et al., PRC 80, 034004 (2009); Lazauskas, Song, Park, arXiv:0905.3119

Electron-deuteron observables

$$
\begin{aligned}
G_C &= \frac{1}{3|e|}\left(\langle 1|J^0|1\rangle + \langle 0|J^0|0\rangle + \langle -1|J^0|-1\rangle\right), \\
G_Q &= \frac{1}{2|e|\eta M_d^2}\left(\langle 0|J^0|0\rangle - \langle 1|J^0|1\rangle\right) \\
G_M &= -\frac{1}{\sqrt{2\eta}|e|}\langle 1|J^+|0\rangle; \qquad \eta = \frac{Q^2}{4M_d^2}
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THEORY

EXPERIMENT

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left[A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right]; \qquad T_{20}(Q^2;\theta_e)$$

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THEORY

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\begin{aligned}
A &= G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 M_d^4 G_Q^2, \\
B &= \frac{4}{3}\eta(1+\eta)G_M^2, \\
T_{20} &= -\frac{1}{\sqrt{2}}\frac{1}{A(Q^2)+B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)}\left[\frac{8}{3}\eta G_C(Q^2)G_Q(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)\right. \\
&\qquad \left. + \frac{1}{3}\eta\left\{1+2(1+\eta)\tan^2\left(\frac{\theta_e}{2}\right)\right\}G_M^2(Q^2)\right].
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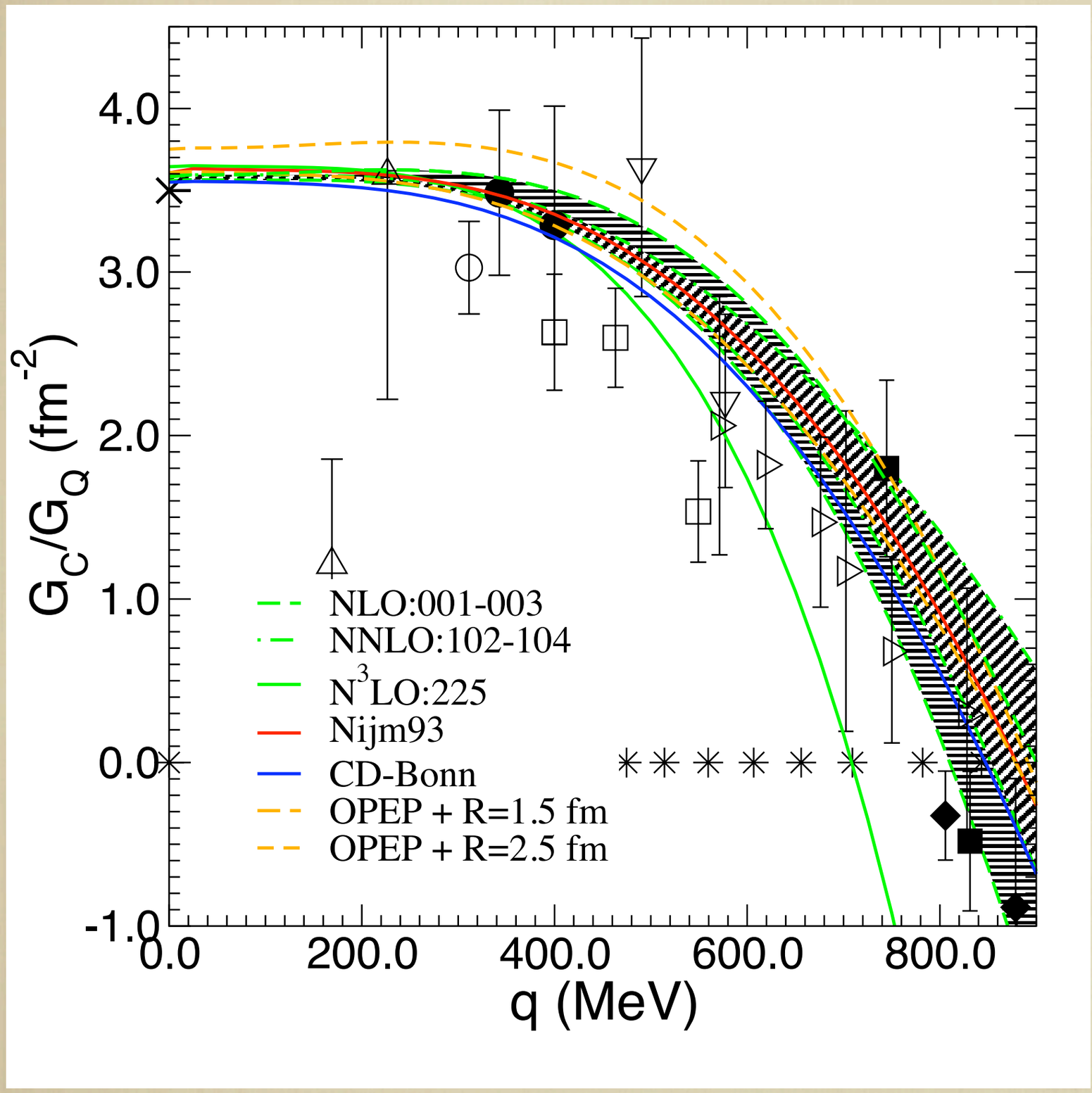
G_C/G_Q at NNLO and beyond



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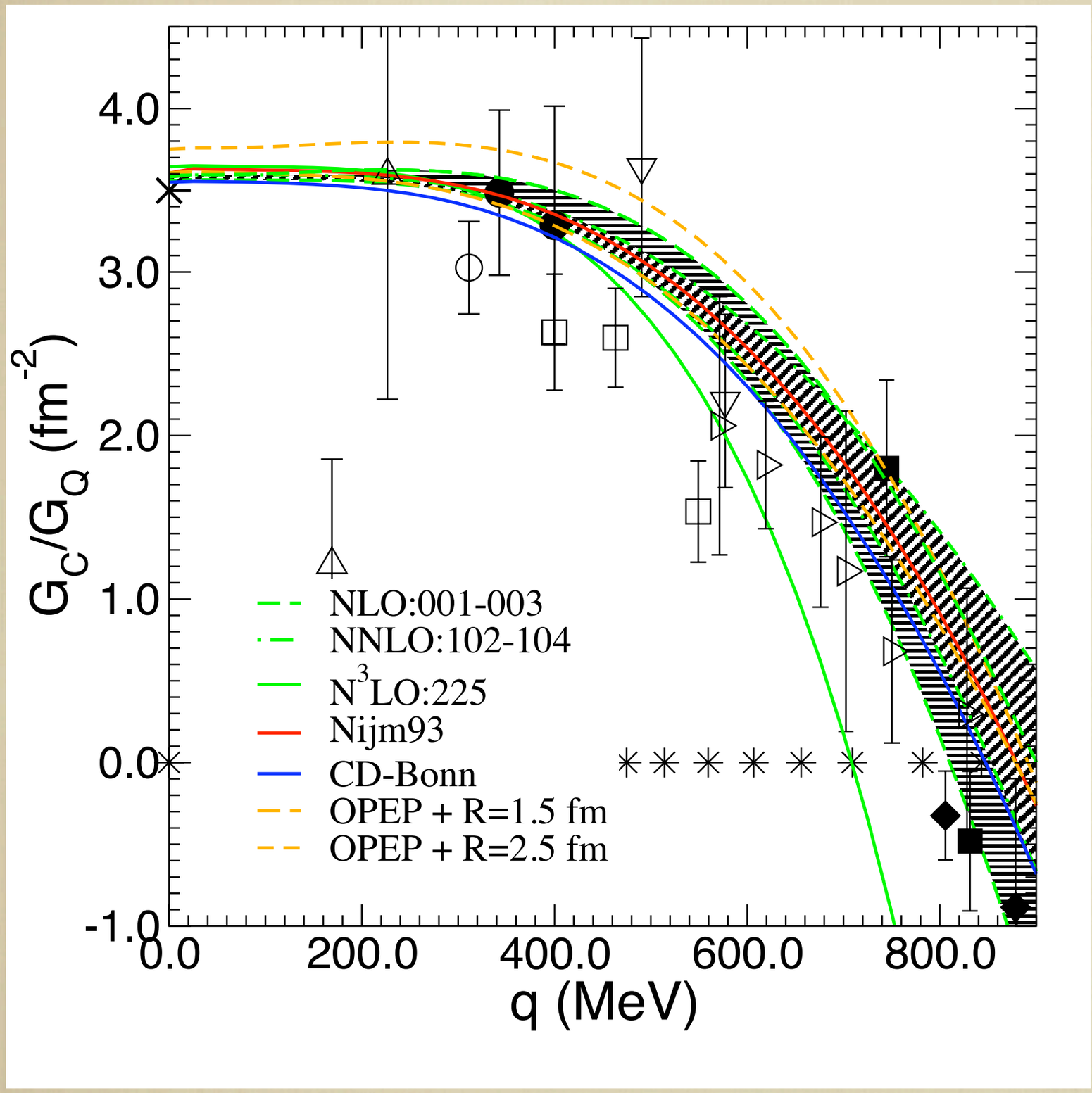

- Adjust O(eP^5) contact term to reproduce Q_d: predict ratio

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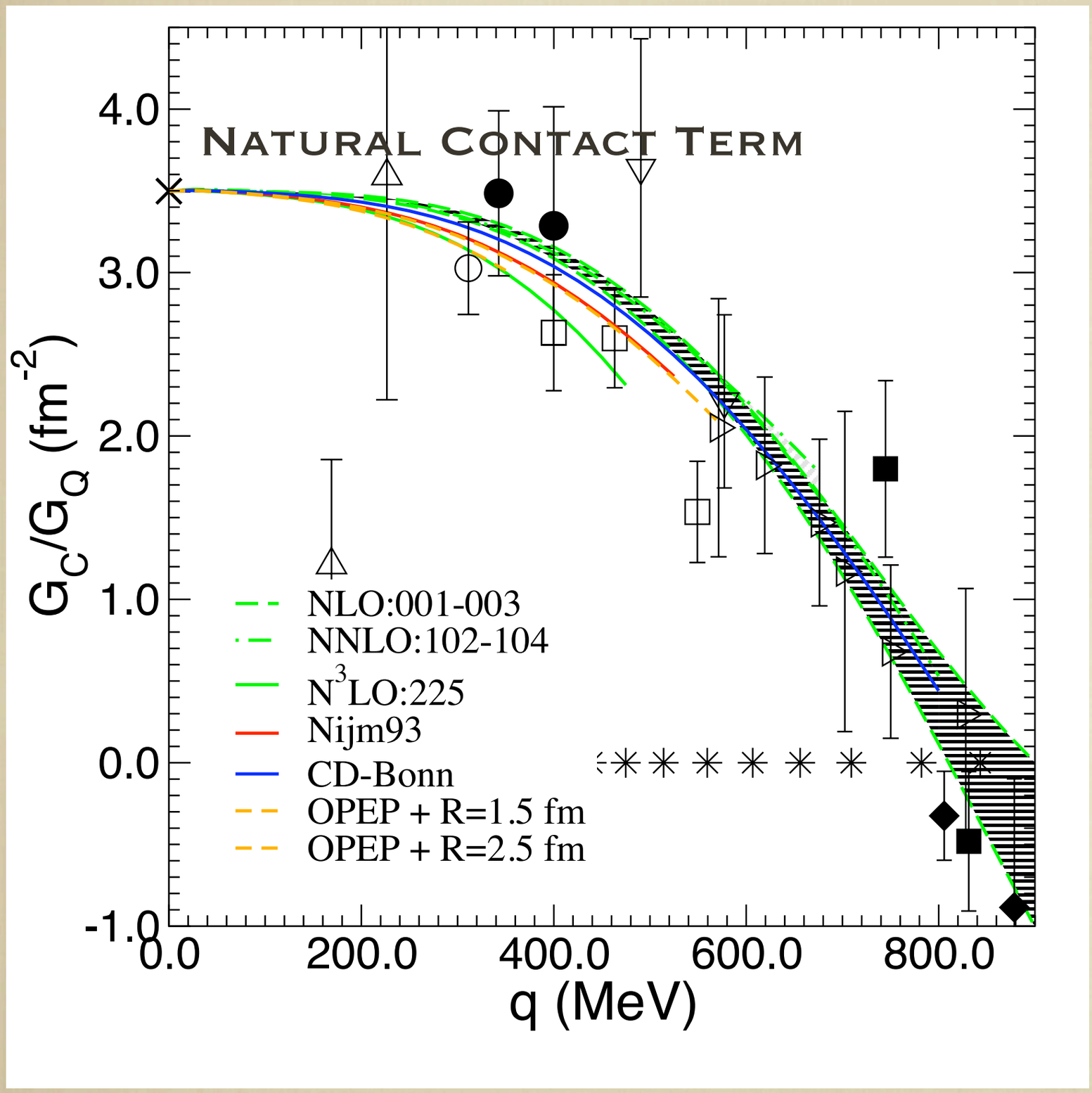

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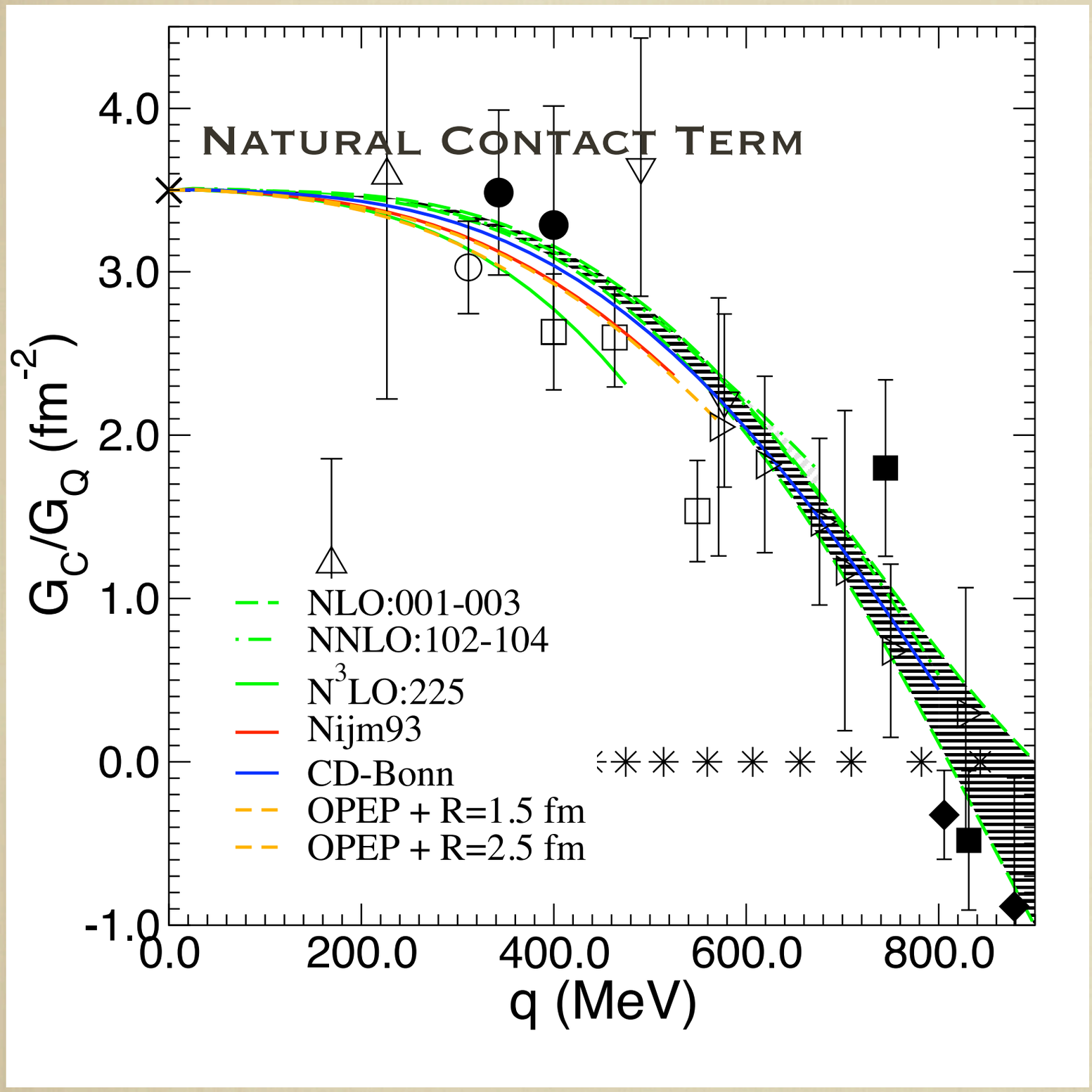
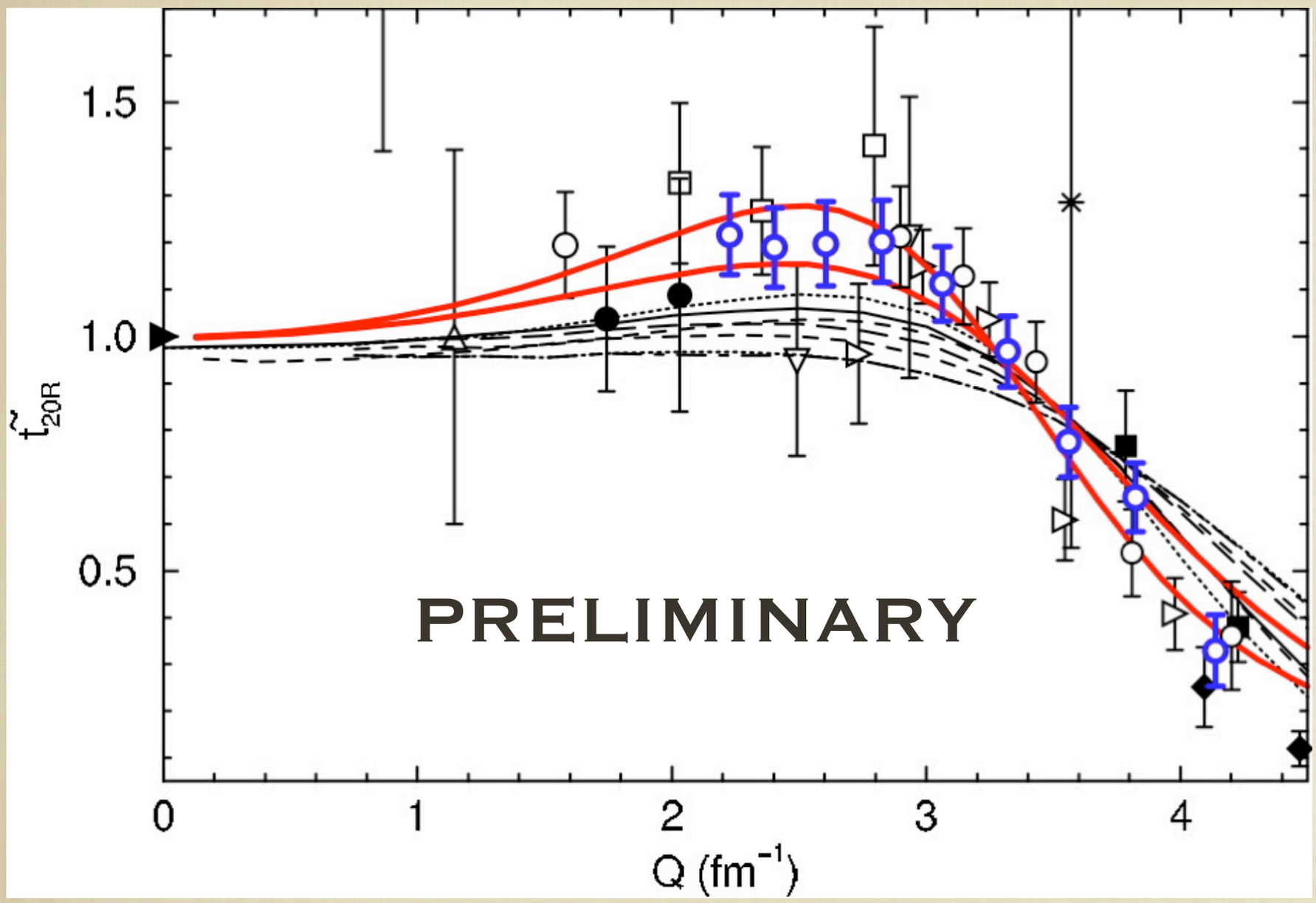


- Adjust O(eP^5) contact term to reproduce Q_d: predict ratio
- Ratio largely independent of short-distance physics for q<600 MeV
- G_C/G_Q to 3% at q= 0.39 GeV

BLAST data on t_{20}

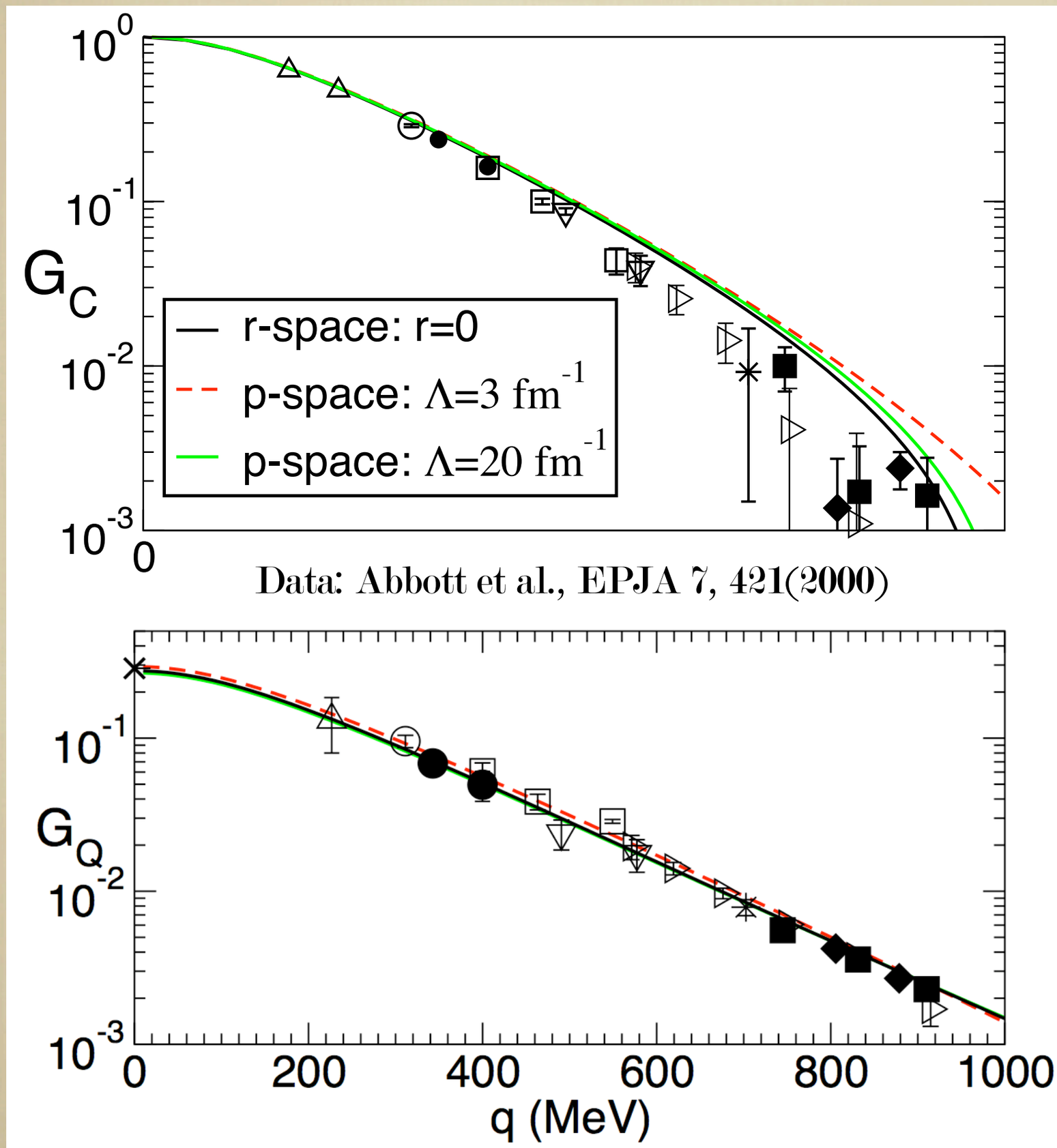


FIRST TWO POINTS GIVE NORMALIZATION

Courtesy M. Garcon

$$\tilde{t}_{20\,R} = -\frac{3\tilde{t}_{20}}{\sqrt{2}Q_d Q^2} \quad \leftrightarrow \quad G_C/G_Q$$

Results for G_C and G_Q at LO



Pavon Valderrama, Ruiz Arriola, Nogga, DP, arXiv:0711.4785

Nucleon form factors included via:

$$\frac{G_C}{G_E^{(s)}} = \langle\psi|e|\psi\rangle + O(P^2)$$

Including TPE in |ψ> improves description

Successes in A=2

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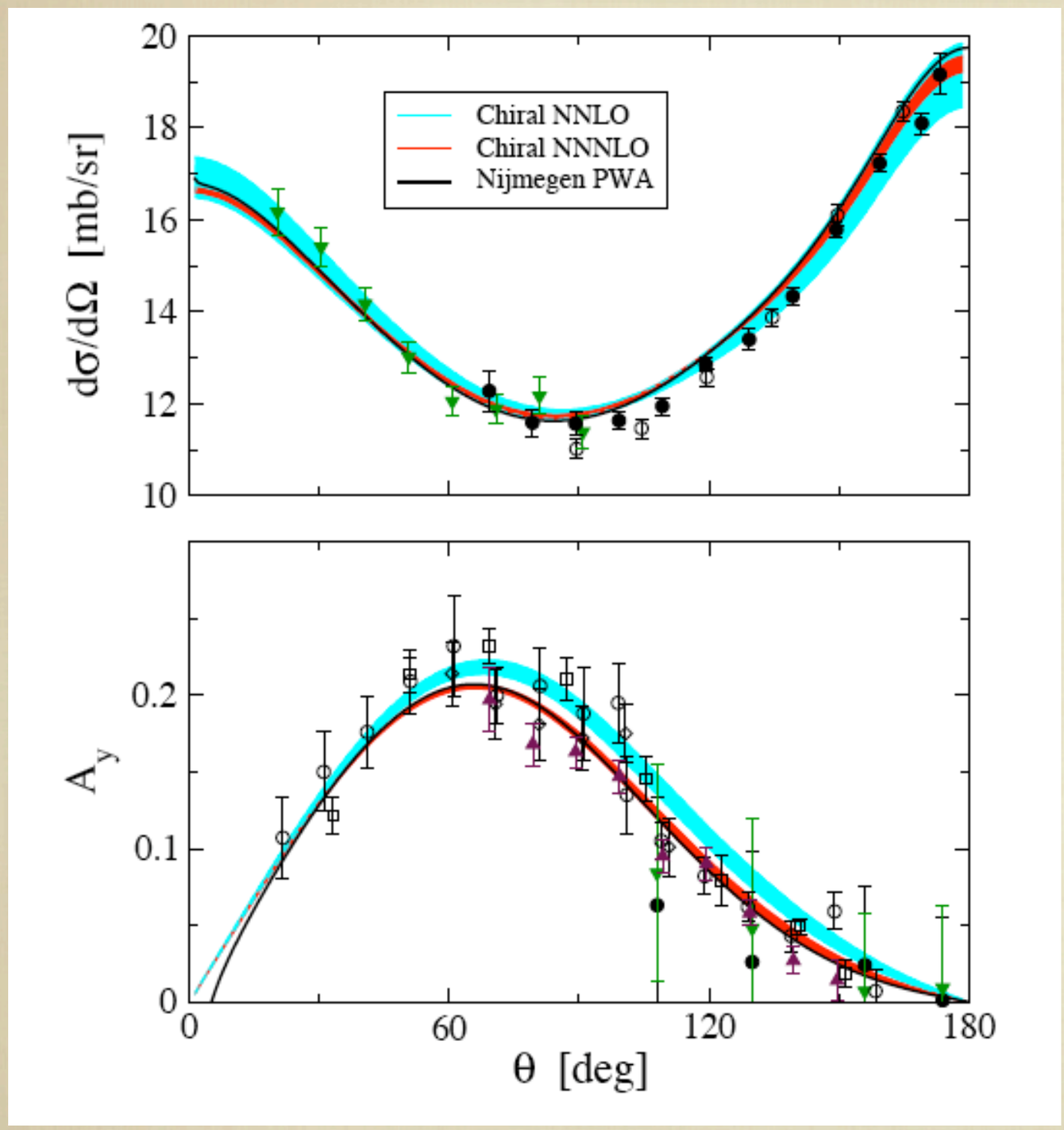

- Example at E_{lab}=50 MeV

Epelbaum, Meissner, Gloeckle, Nucl.Phys.A747, 362-424 (2005)

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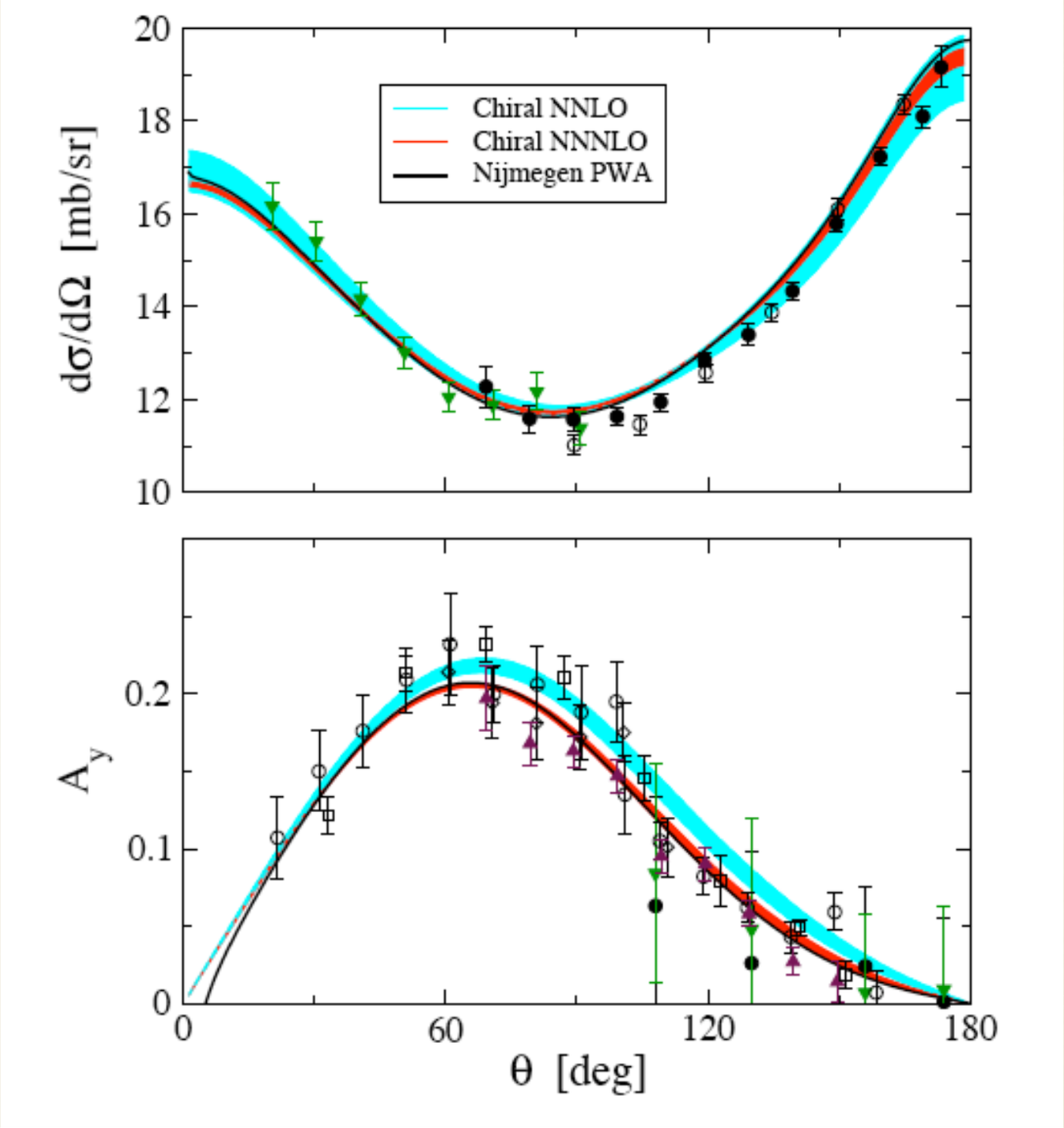

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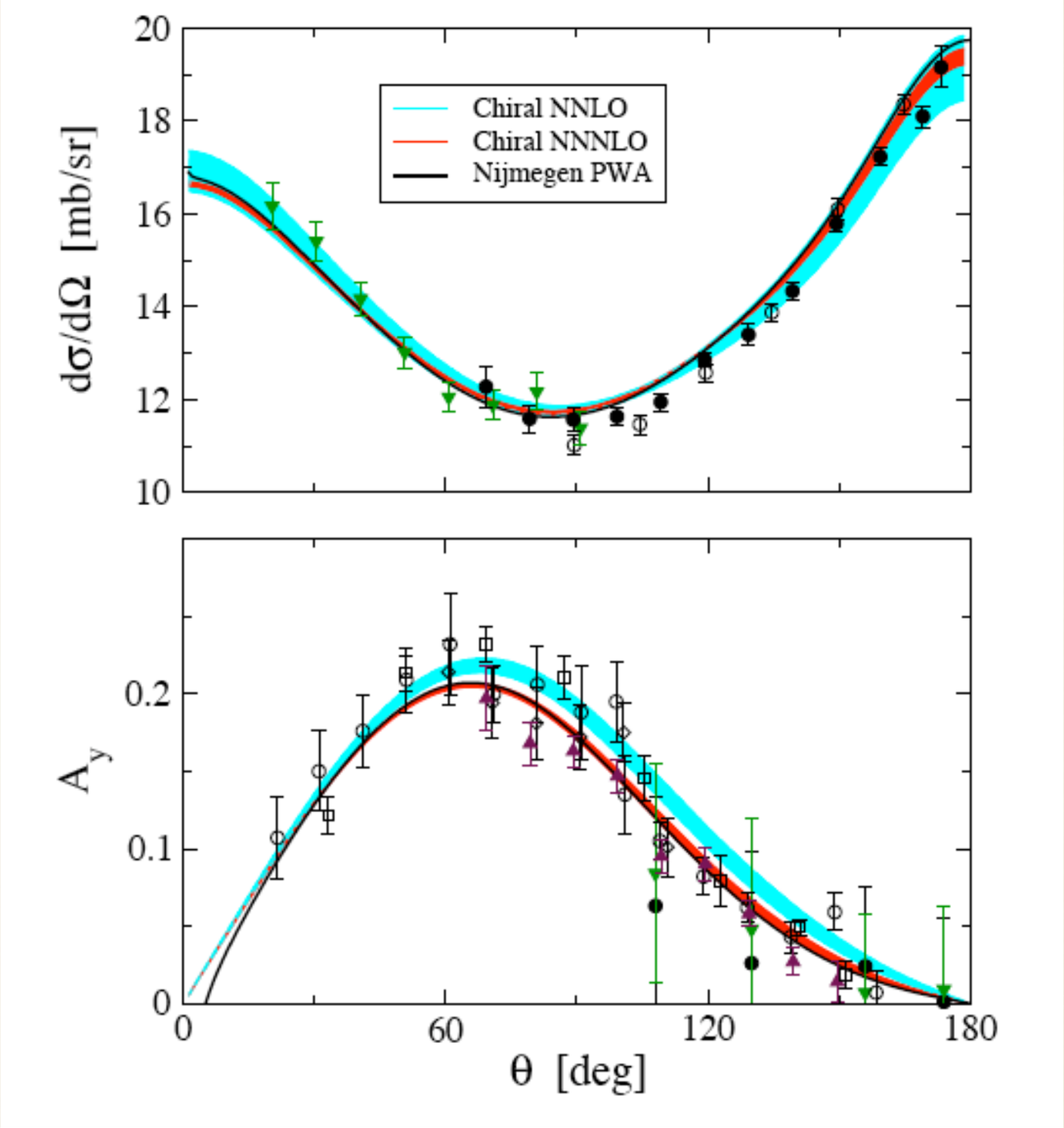

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- Entem/Machleidt: N^3LO potential, χ^2/dof comparable to AV18.
- Nijmegen phase-shift analysis + NN data gives: $f^2_{\pi NN}$=0.0750(9), $m_{\pi+}$=139.6(1.3) MeV
- χPT TPE in pp PSA gives c_i's, m_π=128(9) MeV

Epelbaum, Meissner, Gloeckle, Nucl.Phys.A747, 362-424 (2005)

Rentmeester et al., 1999